Second midterm of Math 54, Fall 2012

Oct 25th 2012

Name:

SID:

Section and GSI name:

Problem	Points Earned	Total Points
1		30
2		15
3		20
4		10
5		25
Total		100

(*30 points total*) Consider the matrix

$$A = \begin{bmatrix} 1 & a & 1 \\ 0 & -1 & b \\ 0 & 0 & 1 \end{bmatrix}.$$

1. (*10 points*) State all necessary conditions, if any, on a, b for A to be invertible. Justify your answer.

2. (*20 points*) State all necessary conditions, if any, on a, b for A to be diagonalizable. Justify your answer.

(*15 points*) Find the closest point to $\mathbf{y}$ in the subspace W of $\mathbb{R}^3$ spanned by $\mathbf{u}_1$ and $\mathbf{u}_2$:

$$\mathbf{y} = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}, \qquad \mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \qquad \mathbf{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}.$$

(*20 points*) Find any orthonormal basis for the inner product on $\mathbb{P}_2(x)$ - i.e. polynomials in x of degree up to 2 - given by

$$\langle p(x), q(x) \rangle = \int_0^1 p(x)q(x)\,dx.$$

(*10 points*) Let $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Find a useful expression for the matrix for A^k, where k is an arbitrary integer > 0. Hint: Use eigenvalues and eigenvectors.

(*25 points total*)

1. (*15 points*) Find the equation $y = \alpha_0 + \alpha_1 x$ of the least square line that best fits the data points $(2,1), (5,2), (7,3), (8,3)$.

2. (*10 points*) For the same data points consider the equation $y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^4$ of the best fit least square degree four polynomial. True or false: this has a unique solution. Justify your answer.