MATH 54 – MIDTERM #2

Problem #1. (a) Compute the Wronskian of the functions $y_1(x) = e^x$ and $y_2(x) = \sin x$.

(b) Explain why it is impossible that y_1 and y_2 are both solutions of a second–order linear ODE

$$y'' + p(x)y' + q(x)y = 0 \qquad (0 \leq x \leq \pi),$$

where p and q are continous functions.

Problem #2. Find the determinant of the matrix

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 7 & 1 & 4 & 1 \\ 1 & 0 & 6 & 0 \\ 0 & 3 & 6 & 2 \end{bmatrix}.$$

Problem #3. Let

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

Find an invertible matrix S such that

$$S^{-1}AS = \Lambda \quad \text{is diagonal.}$$

Problem #4. Let A be an $n \times n$ *symmetric* matrix.

Prove that if $\mathbf{v}_1, \mathbf{v}_2$ are eigenvectors of A corresponding to the *distinct* eigenvalues λ_1, λ_2, then

$$\mathbf{v}_1, \mathbf{v}_2 \quad \text{are orthogonal.}$$

Problem #5. An $n \times n$ matrix B is called *skew-symmetric* if

$$B^T = -B.$$

Show that if n is odd and B is skew-symmetric, then B is not invertible.

(Hint: use determinants.)