MATH 54 – MIDTERM #1

Problem #1. Find the inverse of the matrix

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 3 \\ 0 & 1 & 0 \end{bmatrix}.$$

Problem #2. Compute the rank of the following matrix and find bases for its row space, column space and null space:

$$\begin{bmatrix} 1 & 2 & -1 \\ 1 & 9 & -1 \\ -3 & 8 & 3 \\ -2 & 3 & 2 \end{bmatrix}.$$

Problem #3. The *Cauchy-Schwarz inequality* states that

$$|\mathbf{x} \cdot \mathbf{y}| \leq ||\mathbf{x}||||\mathbf{y}|| \qquad \text{for vectors } \mathbf{x}, \mathbf{y} \in \mathbb{R}^n.$$

Use this to prove the *triangle inequality*

$$||\mathbf{x} + \mathbf{y}|| \leq ||\mathbf{x}|| + ||\mathbf{y}|| \qquad \text{for vectors } \mathbf{x}, \mathbf{y} \in \mathbb{R}^n.$$

Problem #4. Let A be an $n \times n$ matrix satisfying

$$A^3 = O.$$

Find numbers r and s so that $B = I + rA + sA^2$ solves the equation

$$B^2 = I + A.$$

Problem #5. Suppose that $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ is a basis of $\mathbb{R}^n$ and that A is an *invertible* $n \times n$ matrix.

Prove that $\{A\mathbf{v}_1, \ldots, A\mathbf{v}_n\}$ is a basis of $\mathbb{R}^n$.