PART A: Basics (15 points)

1. **(1 point)** Suppose that `u = [-3, 0, 0, 2, 5]` and `v = [-3, -2, 0, 2, 4]`. What are the results of the following operations?

```
(a)   u > 0

(b)   v == u
```

2. **(1 point)** Consider the following MATLAB code that evaluates `y` for a predefined vector, `x`.

```
clear y
for i = 1:length(x)
   j = 0;
   if x(i) == 0 | x(i) < 0
      j = 1;
   end
   y(i) = j;
end
```

Write a single MATLAB statement to accomplish the above.

»

3. **(1 point)** When the following MATLAB program is executed what would be the output from the last statement?

```
p=0; q=0; r=0;
for i = 1 : 2 : 3
      p = p + 1;
      for j = 1 : i
         q = q + 1;
      end
end
disp([p,q])
```

4. **(1 point)** Given that `wt = [2, 5, 6, 8, 3];` what will be the value of num after MATLAB executes the following set of command lines?

```
num = 0;
for i = 1 : length(wt)
   if (wt(i) <= 5)
      num = num + 1;
   end
end
disp(num)
```

5. **(1 point)** Write a single MATLAB statement to accomplish exactly the same as the code in the above problem.

»

6. **(4 points)** If `p = 3.5, q = 2,` and `k = -5.5,` what will be the result of the following expressions?

(a) `(p + q) >= k` ____ ________

(b) `(p < 5) & (k > 0)` ____ ________

(c) `(p > q) | (p < q)` ____ ________

(d) `p + (q >= k)` ____ ________

7. **(4 points)** Given below are two incomplete `for` loops. Fill in the blanks of the MATLAB code such that the first loop is equivalent to the command `B = sum(A)`, and the second loop is equivalent to `C = sum(A')`, where A is a predefined (m x n) matrix.

```
[m, n] = size(A);
for j= _     ___ : 1 : _     __
   B(j) = sum( _____A(:,j)________________ )
end
for i= _     _____ : 1 : __     __
   C(i) = sum( ___               __________ )
end
```

8. **(2 points)** The height *h* of an object as a function of time *t* is described by the equation

$$h(t) = 20 + 10\,t - 2\,t^{2.3}$$

Solution to this equation for $h(t) = 0$ is known to exist in the neighborhood of $t = 3$. (Fill in the following blank)

```
h = @(t)
```

In one line, write a MATLAB instruction which uses the previous handle function to solve the equation $h(t) = 0$

```
>> ____                    ____________________
```

PART B: Functions (10 points)

1. **(6 points)** Fill in the blanks in the following MATLAB function `leap` which returns a `value` of 1 (true) or 0 (false) depending on whether the input argument `year` is a leap year or not. Recall that, by definition, a leap year is a multiple of 4 (but if it is a multiple of 100, it must also be a multiple of 400). Accordingly, note that the years 2000 and 1776 are leap but 1993 and 1900 are not. Hint: The MATLAB function `rem(m,n)` gives the remainder when `m` is divided by `n`.

```
function value = __          __( __          __ )

if ______________________________________
  value = 1;
   if rem(year,100)== 0
    if ____                    ________________
      value = _   _;
    else
      value = _   _;
    end
   end
else
  value = _   _;
end
```

2. A solution is defined as *acid* when its pH is below 7, *neutral* when its pH is exactly 7, and *base* when its pH is above 7.

(a) **(2 points)** Write a MATLAB function called `PH` that receives a single pH value as input argument and returns a single string `'acid'` or `'neutral'` or `'base'` as its output.

(b) **(2 points)** Suppose you are given a set of pH values `val =[5.1, 7.0, 11, 12.5, 6]`. Write a `for` loop to call the `PH` function to determine the corresponding acidity (i.e., `acid`, `neutral` or `base`).

PART C: Linear Algebra (12 points)

1. **(3 points)** Show that the following set of linear equations has no solution:

$$-4x+5y=10$$
$$12x-15y=8$$

a. What is the determinant of the coefficient matrix of these equations? (Fill the four blanks in the matrix A and the result)

b. What is rank(A)?

c. What is the rank of the augmented matrix? (Fill the six blanks in the matrix [A,b] and the result)

where *b* is the right-hand side vector.

2. **(6 points)** Answer the following questions:

(a) What MATLAB function calculates the determinant of a matrix A?

(b) Does the matrix A above have to be square? (Circle the right answer)

YES **NO**

(c) What does the backslash (\) operator do for an (n x n) invertible system of linear equations?

(d) What MATLAB function calculates the inverse of a matrix?

(e) Does the matrix to be inverted have to be square? (Circle the right answer)

YES **NO**

(f) Write a single MATLAB statement to solve the system of equations:

$$\boldsymbol{A\,x^2 = b}$$

where A is a (n x n) square matrix of rank n, x and b are vectors of size $n\times1$. A and b are given. In Matlab notation, $x^2 = x.*x$.

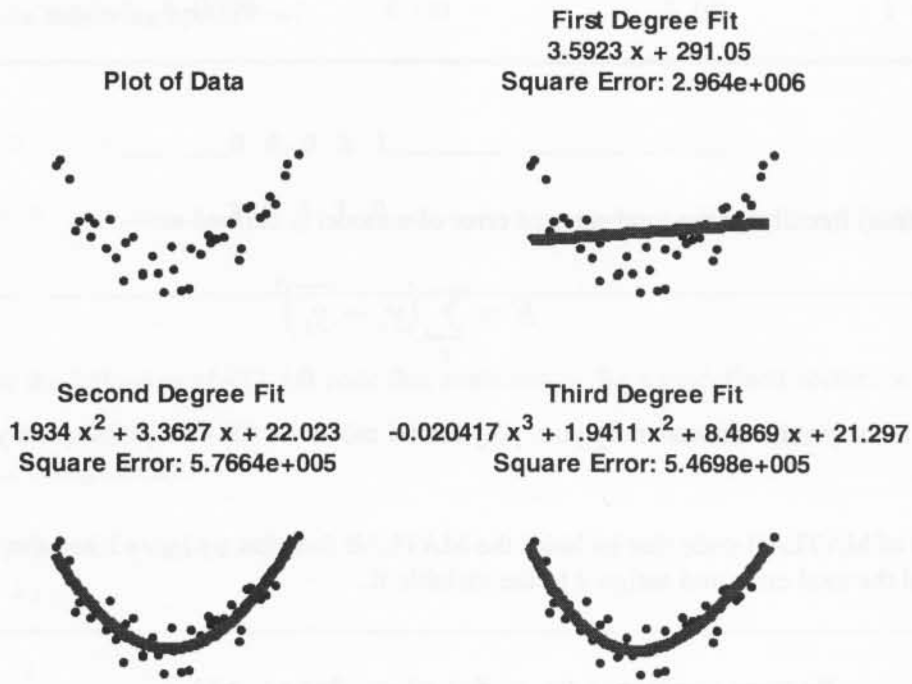
`Find x:`

3. **(3 points)** For what value of a will the following set have a solution in which both x and y are nonzero? Find the relationship between x and y.

$$4x - ay = 0$$
$$-3x + 6y = 0$$

PART D: Least Squares Regression (8 points)

Refer to the figure for the following question.



1. **(1 points)** With the information above, the underlying phenomenon the data is extracted from is most closely approximated by (Circle the right answer):

(a)	A Linear Model	(b)	A Quadradic Model
c)	**A Cubic Model**	(d)	A Non-Polynomial Model

2. **(2 points)** Assume you have variables in the MATLAB workspace called `x_data` and `y_data` which are arrays of data points taken from an experiment. You have reason to believe the data corresponds to a 4^{th} order polynomial model. In one line, use the MATLAB function `polyfit` to find the parameters associated with your model and assign the parameters to a variable `P`.

```
>> ________                                        ________________
```

3. **(1 point)** Assuming you have created the variable `P` in the previous question correctly, how many elements will the array, `P`, have? (Circle the right answer)

(a) 1	(b) 2	(c) 3	(d) 4	**(e) More than 4**

4. **(4 points)** Recall that the total squared error of a model is defined as:

$$E = \sum_i \left(y_i - \overline{y_i}\right)^2$$

where $\overline{y_i}$ is the y value estimated by your polynomial model for the data point x_i and y_i is an individual y data point.

In one line of MATLAB code that includes the MATLAB function `polyval` and the variable `P`, created in D.1, find the total error and assign it to the variable `E`.

```
>> ___                                    ________________
```