

Midterm 1

Monday, February 22, 2010

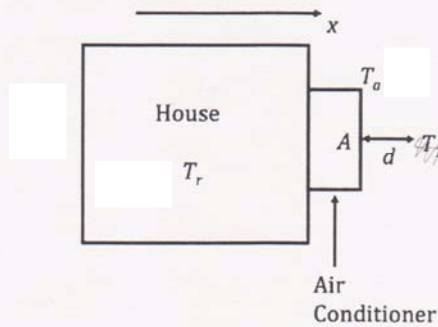
6:00- 8:00 PM

Total Points: 100 + 10

Note: You are allowed one formula card (3.5 " by 5 ", double sided). No calculators or any other electronic devices are permitted.

1. (20 pts) The volume coefficient β is defined as $\beta = (1/V)(dV/dT)$. Calculate β as a function of temperature for the air in a hot balloon. Treat the air as an ideal gas. Assume that the air in the balloon does not escape and the heating process is slow enough to be considered quasistatic.

2. (30 pts) In the summer an air conditioner keeps the temperature of a house at T_r . Assume the outside temperature is $T_1 > T_r$. The temperature of the warm side of the air conditioner is $T_a > T_1$. Assume the air conditioner has perfect Carnot efficiency.



a) Suppose the air at a distance d from the air conditioner cools to the ambient temperature, T_1 (see figure), and the surface area of the warm side of the air conditioner is A . The conductivity of air is k . You may assume heat conduction only takes place in the x -direction (In reality there will be heat conduction in other directions. We are simply trying to make a reasonable estimate in this problem.) Calculate the amount of heat dumped outside per unit time, dQ/dt .

b) Find the amount of power that needs to be supplied to the air conditioner.

e) In the winter, if the temperature of the room is still at T_r and the outside temperature is $T_2 < T_r$ find the heat loss rate, dQ'/dt , for the house.

3. (15 pts) The coefficient of performance, CP, for a refrigerator is defined as $CP = |Q_L|/W$. During one cycle of the operation of a real refrigerator, $|Q_L|$ amount of heat is absorbed from a low temperature reservoir at T_L and $|Q_H|$ amount of heat is released to a high temperature reservoir at T_H . Its CP is only a fraction $f (< 1)$ of CP_{ideal} for a Carnot refrigerator. Find the total entropy change at the end of the cycle in terms of Q_H , T_H and f . You may assume Q_H is positive and don't have to write the absolute signs in your calculation.

4. (25 pts) An n -mole sample of an ideal diatomic gas at a pressure of P_1 and temperature of T_1 undergoes a process in which its pressure increases linearly with temperature. The final temperature and pressure are T_2 and P_2 . (*Hint: Is volume necessarily a constant in this process?*)

a) Determine the change in internal energy. (Assume five active degrees of freedom.)

b) Determine the work done by the gas.

c) Determine the heat added to the gas.

5. (10 pts + 10 Bonus pts) Light can sometimes be treated as particles, called photons. Photons can form a "gas". The energy density of a photon gas is given by

$$\frac{U}{V} = Ak_B^4 T^4$$

where A is a constant. This is closely related to the Stefan-Boltzmann equation in the textbook.

a) (10 pts) Find dS in terms of T and V for the photon gas by using the first law of thermodynamics and considering the isochoric process.

b) (*Bonus*: 5 pts) Integrate the expression for dS and find S as a function of T . You may set the integration constant to zero.

c) (*Bonus*: 5 pts) Find the condition that V and T have to satisfy if a photon gas expands adiabatically (or isentropically). Suppose that the expansion of the universe is an adiabatic process and that we can ignore all other contents of the universe except photons. Given that the temperature of the cosmic microwave background radiation today is approximately 3 K, what was the size of the universe when the photon temperature was at 3000 K, compared to its size today? If we further assume that the size of the universe, a , expands as $a(t) = Bt$, where B is a constant and t is time, at what fraction of the present age of the universe was its temperature at 3000 K?

The End