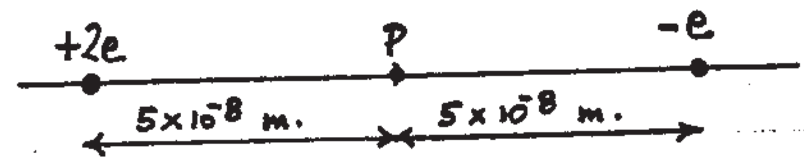
Physics 57B <u>Solutions to Second Exam</u> Aug. 3, 1992

(1)



V_1 (due to α) $= +\dfrac{Q}{4\pi\epsilon_0 r} = \dfrac{(2)(1.6\times10^{-19})}{(4\pi)(8.85\times10^{-12})(5\times10^{-8})} = +0.0575$ volts

V_2 (due to β^-) $= \dfrac{-Q}{4\pi\epsilon_0 r} = \dfrac{(-1.6\times10^{-19})}{4\pi(8.85\times10^{-12})(5\times10^{-8})} = -0.0288$ volts

At point P, total potential $V = V_1 + V_2 \Rightarrow \boxed{V = +0.0287 \text{ volts}}$

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(2) When switch is at A, capacitor is charging through resistor R with time constant $RC = (10^7)(10^{-9}) = 10^{-2}$ seconds. In 12 hours, capacitor will reach its final charge $Q(\text{final}) = C\mathcal{E} = 3\times10^{-9}$ Coul.

When switch is moved to B, capacitor <u>discharges</u> its charge of $q(0) = 3\times10^{-9}$ C. at $t=0$. We know the charge $q(t)$ at time t is

$$q(t) = q(0)e^{-t/RC} \Rightarrow I(t) = \frac{dq}{dt} = q(0)\left(\frac{-1}{RC}\right)e^{-t/RC}$$

For $I = 1\times10^{-7}$ Amps, we have

$$(1\times10^{-7}) = (3\times10^{-9})\left(-\frac{1}{10^{-2}}\right)e^{-t/0.01} \Rightarrow$$

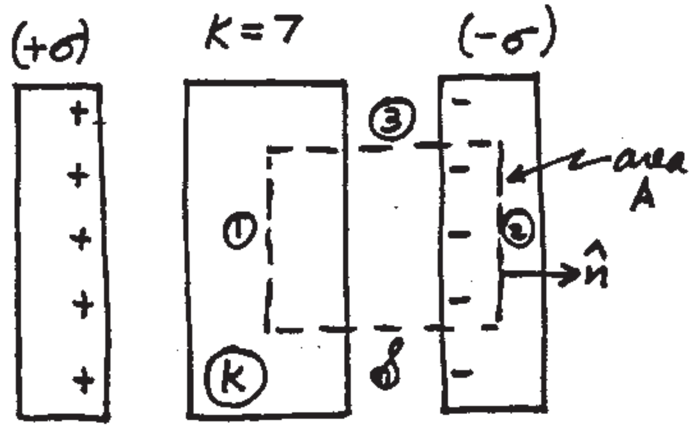
$$(1\times10^{-7}) = (-3\times10^{-7})e^{-t/0.01}$$

$$-0.3333 = e^{-t/0.01} = e^{-100t} \Rightarrow \ln|-0.3333| = -100t$$

$$\ln(0.3333) = -100t \Rightarrow -1.099 = -100t$$

$$\boxed{t = 1.099\times10^{-2} \text{ sec}}$$

(3)



$$\sigma = \frac{8.9 \times 10^{-10}\ C.}{10^{-2}\ m^2}$$

$$\sigma = 8.9 \times 10^{-8}\ Cm^{-2}$$

Draw cylindrical Gaussian surface $\mathcal{S}$ shown dotted.

<u>Gauss Law:</u> $K\epsilon_0 \oint\!\!\oint_{\mathcal{S}} \underline{E} \cdot \hat{n}\, dA = q(\text{free})$ inside $\mathcal{S}$

No electric flux through surface ③; no <u>free</u> charge inside surface ① in the dielectric. Free charge (σA) inside surface ② inside metal; Since $\underline{E} \parallel \hat{n}$, Gauss' Law becomes

$$K\epsilon_0 E \oint\!\!\oint_{\mathcal{S}} dA = \sigma A \implies K\epsilon_0 E A = \sigma A \implies \boxed{E = \frac{\sigma}{K\epsilon_0}}$$

$$E = \frac{8.9 \times 10^{-8}}{7(8.85 \times 10^{-12})} = 1436 \text{ Volts } m^{-1} \qquad \boxed{E = 1436\ Vm^{-1}}$$

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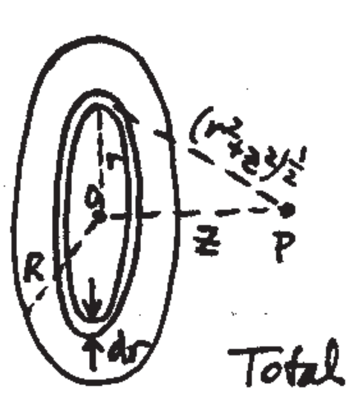
(4) Divide disc into annular rings of radius r and area $dA = 2\pi r\, dr$ where $0 \leq r \leq R$ and $z = $ constant. Charge dq on annular ring is $dq = \sigma dA = 2\pi\sigma r\, dr$. At point P, potential dV due to dq is



$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{(r^2+z^2)^{1/2}} = \frac{1}{4\pi\epsilon_0} \frac{2\pi\sigma r\, dr}{(r^2+z^2)^{1/2}}$$

Total potential $V = \int dV = \int_0^R dV$

$$V = \frac{2\pi\sigma}{4\pi\epsilon_0} \int_0^R \frac{r\, dr}{(r^2+z^2)^{1/2}} = \frac{\sigma}{2\epsilon_0}\left[\sqrt{r^2+z^2}\right]_0^R \implies \boxed{V(z) = \frac{\sigma}{2\epsilon_0}\left[\sqrt{R^2+z^2} - z\right]}$$

(5) From lecture, we know that the potentials on the surfaces of the two spheres are

(a) $V(r) = \frac{Q}{4\pi\epsilon_0 r}$; $V(R) = \frac{Q}{4\pi\epsilon_0 R}$

Since $R > r$, $V(R) < V(r)$. This means that the electric potential energy of a mobile (+) charge is less on the surface of the larger sphere of radius R. Positive charge therefore flows from higher potential energy (on the smaller sphere of radius r) toward the larger sphere (of radius R) where the electric potential energy is smaller.

(b) The transfer of charge continues until the potential energy of the charge on the two spheres is the same. This means the transfer stops when the potential is the same on the two spheres. If q is amount of charge transferred,

$$V(r) = \frac{(Q-q)}{4\pi\epsilon_0 r} \; ; \; V(R) = \frac{(Q+q)}{4\pi\epsilon_0 R}$$

When $V(r) = V(R)$, we have

$$\frac{(Q-q)}{4\pi\epsilon_0 r} = \frac{(Q+q)}{4\pi\epsilon_0 R} \Longrightarrow R(Q-q) = r(Q+q)$$

$$RQ - Rq = rQ + rq \Longrightarrow Q(R-r) = q(R+r)$$

$$\boxed{q = Q\left(\frac{R-r}{R+r}\right)}$$