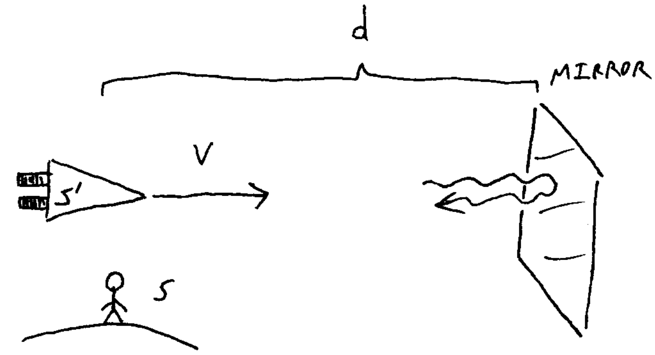
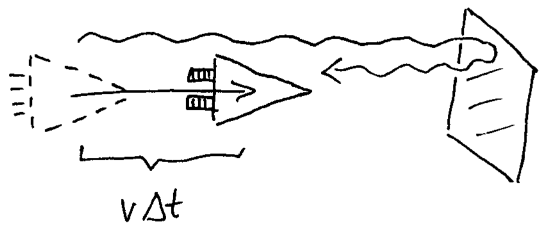
PHYSICS 7C
SPRING '07
LEE MT #2
PROBLEM #2



a)



ROCKET MOVES $v\Delta t \Rightarrow$ LIGHT MOVES $d + (d - v\Delta t) = 2d - v\Delta t$

$$c\Delta t = 2d - v\Delta t$$

$$\boxed{\Delta t = \frac{2d}{c+v}}$$

b) TWO EVENTS: 1. LIGHT EMITTED BY S'
2. LIGHT INTERCEPTED BY S'

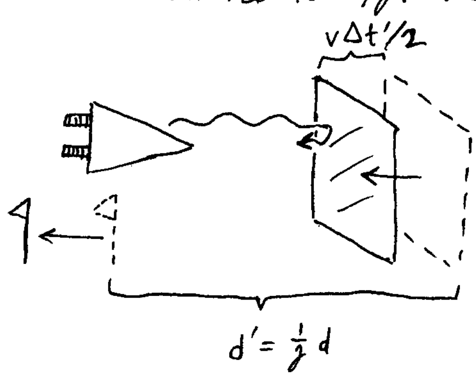
SAME PLACE IN S', SO $\Delta x' = 0$

$$\Delta t = \gamma(\Delta t' + v\Delta x'/c^2) = \gamma \Delta t' \Rightarrow \Delta t' = \frac{1}{\gamma}\Delta t$$ TIME DILATION FORMULA!

$$\Delta t' = \sqrt{1-\frac{v^2}{c^2}}\,\frac{2d}{c+v} = \frac{\sqrt{1-\beta^2}}{1+\beta}\,\frac{2d}{c} = \boxed{\sqrt{\frac{1-\beta}{1+\beta}}\,\frac{2d}{c}}$$

$$\left(\text{-OR-}\quad \Delta t' = \gamma(\Delta t - v\Delta x/c^2) = \gamma\,(\Delta t - v(v\Delta x)/c^2) = \sqrt{1-\beta^2}\,\Delta t = \frac{1}{\gamma}\Delta t\right)$$

ONE CAN ALSO MAKE ARGUMENTS USING LENGTH CONTRACTION. IMAGINE A FLAG FIXED AT THE STARTING POSITION OF S' IN S FRAME. THE FLAG-MIRROR DISTANCE d IS CONTRACTED TO d/γ. WE HAVE THE FOLLOWING SITUATION IN S' FRAME



(DASHED OBJECTS ARE AT $t' = 0$.
SOLID OBJECTS ARE AT $t' = \Delta t'/2$...
HALFWAY THROUGH THE PHOTON'S TRIP, AT THE INSTANT IT BOUNCES.)

TOTAL DISTANCE LIGHT TRAVELS: $2(d' - v\frac{\Delta t'}{2})$

$$\Rightarrow \quad c\Delta t' = 2(d' - v\frac{\Delta t'}{2})$$

$$\Delta t' = \frac{2d'}{c+v} = \frac{1}{\gamma}\frac{2d}{c+v} = \frac{1}{\gamma}\Delta t \quad \checkmark$$

c) SINCE THE MIRROR IS IN S FRAME, THE BOUNCE IN THAT FRAME PRESERVES FREQUENCY AND WAVELENGTH. IT'S THUS EASIEST TO SWITCH FRAMES $S' \to S$, THEN BOUNCE, THEN SWITCH BACK $S \to S'$.

SINCE S' IS MOVING IN THE DIRECTION OF PHOTON, IT WILL APPEAR BLUESHIFTED ACCORDING TO S

$$f = \sqrt{\frac{1+\beta}{1-\beta}}\, f'$$

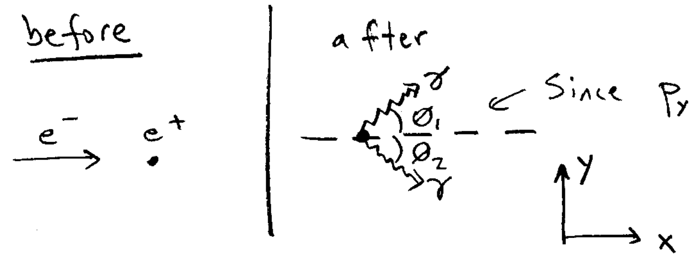
AFTER THE BOUNCE, f IS UNCHANGED

$$f_{REF} = f = \sqrt{\frac{1+\beta}{1-\beta}}\, f'$$

NOW SWITCH BACK. ACCORDING TO S', THE EMITTER (THE MIRROR) IS MOVING IN THE DIRECTION OF THE PHOTON. IT WILL AGAIN BE BLUESHIFTED

$$f'_{REF} = \sqrt{\frac{1+\beta}{1-\beta}}\, f_{REF} = \boxed{\frac{1+\beta}{1-\beta} f'}$$

#3)



← Since $P_y = 0$, $\phi_1 = \phi_2 \equiv \phi$

$$E_i = E_{e^-} + E_{e^+}$$

$$= (K_{e^-} + M_{e^-}c^2) + (M_{e^+}c^2)$$

$$= 1 \text{ MeV} + 2(.511 \text{ MeV}) \cong 2.022 \text{ MeV}$$

$E_f = 2E_\gamma$ & $E_i = E_f \Rightarrow E_\gamma = \frac{E_i}{2} = \boxed{1.011 \text{ MeV}}$ (for each photon)

b.) since $m_\gamma = 0$, $|\vec{p}_\gamma| = E_\gamma / c = \boxed{1.011 \text{ MeV}/c}$

c.) $P_{i,x}c = \sqrt{E_{e^-}^2 - (M_{e^-}c^2)^2}$

$$= \sqrt{(K_{e^-} + M_{e^-}c^2)^2 - (M_{e^-}c^2)^2} = \sqrt{(1 \text{ MeV} + .511 \text{ MeV})^2 - (.511 \text{ MeV})^2}$$

$$\cong 1.4 \text{ MeV}$$

$$P_{f,x} = 2|\vec{p}_\gamma| \cos\phi$$

$$\Rightarrow \phi = \cos^{-1}\left(\frac{P_{f,x}}{2|\vec{p}_\gamma|}\right) = \cos^{-1}\left(\frac{P_{i,x}}{2|\vec{p}_\gamma|}\right) = \cos^{-1}\left(\frac{1.4}{2(1.011)}\right) \cong \boxed{45.3^\circ}$$

#4.) $eV_1 = h\nu_1 - \phi$

$$- \quad eV_2 = h\nu_2 - \phi$$

$$e(V_1 - V_2) = h(\nu_1 - \nu_2)$$

$$\Rightarrow h = \frac{e(V_1 - V_2)}{\nu_1 - \nu_2}$$

$$= e(V_1 - V_2)/[c(1/\lambda_1 - 1/\lambda_2)]$$

$$= 6.624 \times 10^{-34} \text{ J}\cdot\text{s}$$

$$\Rightarrow \phi = h\nu_1 - eV_1$$

$$= 2.24 \text{ eV} \quad (\text{or } 3.6 \times 10^{-19} \text{ J})$$