PHYSICS 7B, Section 1 - Fall 2012
Midterm 2, C. Bordel
Monday, October 29, 2012
7pm-9pm

<u>Integrals and derivatives:</u>

$$\int \frac{dx}{x^2} = \frac{-1}{x};\ \int \frac{dx}{x} = Lnx;\ \int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}};\ \frac{\partial(\tan\theta)}{\partial\theta} = 1+\tan^2\theta = \frac{1}{\cos^2\theta}.$$

<u>Infinitesimal displacement and gradient:</u>

* cylindrical coordinate system

$$\vec{dl} = dr\,\vec{u_r} + rd\theta\,\vec{u_\theta} + dz\,\vec{u_z}\ ;\ \vec{grad}\, f = \left(\frac{\partial f}{\partial r}\right)\vec{u_r} + \left(\frac{1}{r}\frac{\partial f}{\partial \theta}\right)\vec{u_\theta} + \left(\frac{\partial f}{\partial z}\right)\vec{u_z}$$

* spherical coordinate system

$$\vec{dl} = dr\,\vec{u_r} + rd\theta\,\vec{u_\theta} + r\sin\theta d\varphi\,\vec{u_\varphi}\ ;\ \vec{grad}\, f = \left(\frac{\partial f}{\partial r}\right)\vec{u_r} + \left(\frac{1}{r}\frac{\partial f}{\partial \theta}\right)\vec{u_\theta} + \left(\frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\right)\vec{u_\varphi}$$

<u>Problem 1</u> - Linear charge distribution (20 pts)

A semi-circle of radius R is attached at point O to a rod of length 2ℓ, a shown in Figure 1, and each one-dimensional object carries a positive and uniform linear charge distribution λ.

a. Using symmetry arguments, determine the direction of the net electric field created at the center C of the semi-circle and explain the method you are going to use to calculate the electric field and justify your choice.
b. Calculate the electric field created by the semi-circle at point C.
c. Calculate the electric field created by the rod at point C.
d. Can you get the electric potential at point C directly from the electric field at point C? Explain.

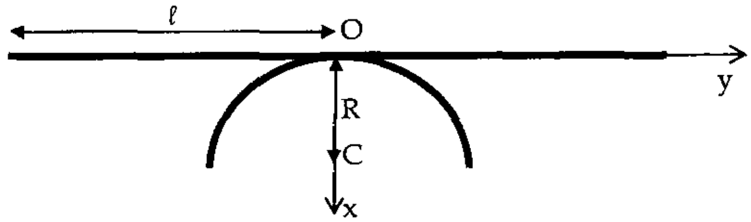

Figure 1

<u>Problem 2</u> – Surface charge distribution (20 points)

We consider two upright infinite coaxial cylinders of radii R_1 and R_2 ($R_1 < R_2$) carrying uniform electric charges, $+\beta$ and $-\beta$ per unit height, respectively. The first cylinder is inside the second one, they have the same symmetry axis, and are very thin shells (see Figure 2).

a. Determine the direction and magnitude of the electric field $\vec{E}$ created by such a system at any point whose distance to the central axis is r. Make sure to show all work, including any relevant drawings.

b. Determine the electric potential $V(r)$ created by this charge distribution at any point whose distance to the symmetry axis is r (regions I, II and III).

   Be sure to state where you are setting electric potential.

c. Make two separate qualitative plots of the functions $E(r)$ and $V(r)$.

d. Calculate the capacitance per unit length of the system.

   Note : If you are stuck, consider the capacitance of a long but finite pair of charged cylinders.

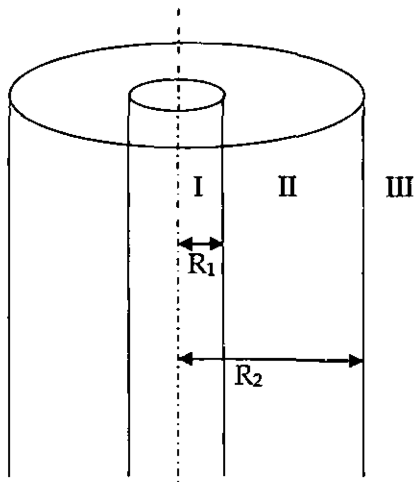

Figure 2

<u>Problem 3</u> - Charged spheres in equilibrium (20 pts)

Two conducting spheres of radii R_1 and R_2 are placed at a distance d ($d >> R_1$ and $d >> R_2$) measured between the two centers. They respectively carry electric charges Q_1 and Q_2 so that the total charge carried by the two spheres is $Q=Q_1+Q_2$ (see Figure 3).

a. As a preliminary step, calculate : (1) the electric potential of an isolated spherical conductor of radius R_a and charge Q_a at its surface; (2) the electric potential created by an isolated charged spherical conductor of radius R_b and charge Q_b at distance $r >> R_b$ from its center.
b. Determine the electric potentials $V_1(Q_1,Q_2)$ and $V_2(Q_1,Q_2)$ as a function of the amount of charge on each sphere.
c. Calculate the electrostatic potential energy of this system of two spheres. *Note*: Make sure you come up with a way to systematically assemble the charges sequentially.
d. We note x the fraction of the total charge carried by the sphere of radius R_1. Determine the value of x that minimizes the electrostatic energy of the system. Give an approximate value of x using the assumptions made on d.

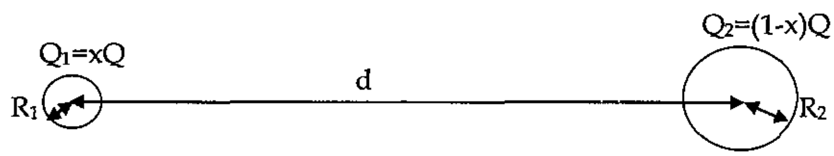

Figure 3

<u>Problem 4</u> - Charge motion in electric field (20 pts)

An electron gun is made of a filament emitting electrons by thermoelectronic effect and two plates that create a potential difference to accelerate the electrons, as shown on Figure 4. The cathode is at potential V_c and the anode is at potential V_a, with $V_a > V_c$. The distance *d* separating the charged plates is assumed to be negligible compared to the size of the plates.

a. What is the magnitude and direction of the electric field between the two plates? Draw it.
b. What is the magnitude and direction of the electric force acting on the electron? Draw it.
c. What is the work done by the electric force in moving an electron from the cathode to the anode?
d. Assuming the electric field is uniform between the two plates, how long does it take for the electron to go from the cathode to the anode?

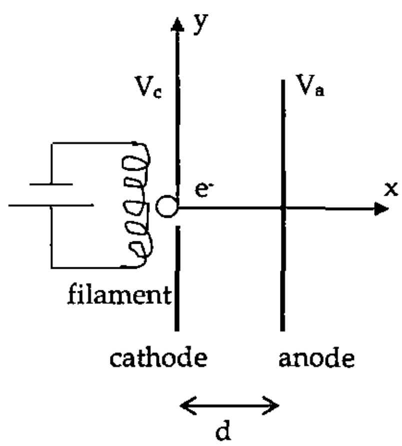

Figure 4

<u>Problem 5</u> - Moving parallel capacitor plates (20 pts)

A capacitor made of two very thin conducting parallel plates of surface area *A* ($A=L^2$), separated by vacuum, has been fully charged. The capacitor remains connected to the battery providing a voltage *V*, but the plates are moved apart (at constant speed) so that the gap increases from ℓ_1 to ℓ_2 ($\ell_1 << L$ and $\ell_2 << L$).

a. Calculate the initial and final electrostatic energies stored by the capacitor.
b. Calculate the force exerted by the negatively charged plate on the other as a function of distance.
c. Calculate the work required to pull the plates apart from ℓ_1 to ℓ_2.
d. Do parts (a) and (c) agree? If not, explain. *Note*: consider the action of the battery.