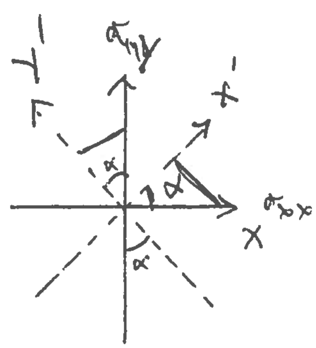
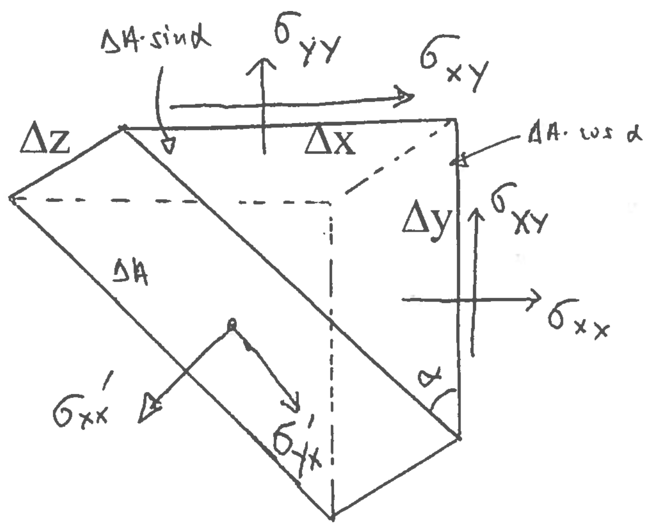
BioE102 Fall 2012
Midterm #1

Instructions: Please write legibly; write your name and SID on the upper right corner of each page.

1. Use the free-body diagram and the balance of stresses to derive σ_{yx}' when the coordinates rotate counterclockwise for an angle α. (30 points)





+15 $\sum F_{y'} = -\sigma_{yx}' \cdot \Delta A + \sigma_{yy} \cdot \Delta A \cdot \sin\alpha \cdot \cos\alpha - \sigma_{xx}\, \Delta A \cdot \cos\alpha \cdot \sin\alpha + \sigma_{xy} \cdot \Delta A \cdot \cos\alpha \cdot \cos\alpha - \sigma_{xy} \cdot \Delta A \cdot \sin\alpha \cdot \sin\alpha$

$\Rightarrow \sigma_{yx}' = (\sigma_{yy} - \sigma_{xx}) \cdot \sin\alpha \cdot \cos\alpha + \sigma_{xy}(\cos^2\alpha - \sin^2\alpha)$ (+ 13)

$\sin(2\alpha) = 2 \cdot \sin\alpha \cdot \cos\alpha$, $\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$, $\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$

$\Rightarrow \boxed{\sigma_{yx}' = \left(\frac{\sigma_{yy} - \sigma_{xx}}{2}\right) \cdot \sin 2\alpha + \sigma_{xy} \cdot \cos 2\alpha}$ (+2)

2. Given σ_{xx}=20 kPa, σ_{yy}=5 kPa and σ_{xy}= 10 kPa at point p, find the values of the principal stresses and the maximum shear stresses. What are the value of α_p and α_s? Draw a 2-D representation of stresses at α_p and α_s. (30 points)

Value α_p, α_s

$$\alpha_p = \frac{1}{2}\tan^{-1}\left(\frac{2\sigma_{xy}}{\sigma_{xx}-\sigma_{yy}}\right) = \frac{1}{2}\tan^{-1}\left(\frac{20}{15}\right) = 26.57^\circ = \alpha_p$$

$$\alpha_s = \frac{1}{2}\tan^{-1}\left(\frac{\sigma_{yy}-\sigma_{xx}}{2\sigma_{xy}}\right) = \frac{1}{2}\tan^{-1}\left(\frac{-15}{20}\right) = 18.43^\circ = \alpha_s$$

+12

Prin. Stresses:

$$\sigma'_{xx,max} = \sigma'_{xx}(\alpha=\alpha_p)$$
$$= \sigma_{xx}\cdot\cos^2(\alpha_p) + 2\cdot\sigma_{xy}\cdot\sin(\alpha_p)\cdot\cos(\alpha_p) + \sigma_{yy}\cdot\sin^2(\alpha_p) = 25\text{ kPa}$$

$$\sigma'_{yy,min} = \sigma'_{yy}(\alpha=\alpha_p)$$
$$= \sigma_{xx}\cdot\sin^2(\alpha_p) + 2\,\sigma_{xy}\cdot\sin(\alpha_p)\cdot\cos(\alpha_p) + \sigma_{yy}\cdot\cos^2(\alpha_p) = 0\text{ kPa}$$

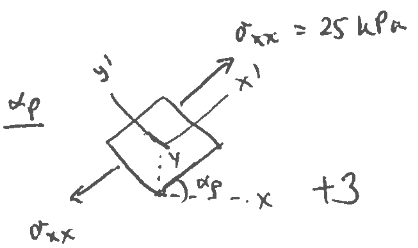
Max Shear Stress:

$$\sigma'_{xy,max} = \sigma'_{xy}(\alpha=\alpha_s) = \sqrt{\left(\frac{\sigma_{yy}-\sigma_{xx}}{2}\right)^2 + \sigma_{xy}^2} = 12.5\text{ kPa}$$

+12

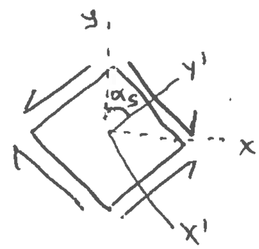
2-D

α_p



+3

α_s



$\sigma'_{xy} = \sigma'_{yx} = 12.5\text{ kPa}$

+3

3. Three strain gauges are used to measure extensional strains ε_{0°, ε_{45° and ε_{90° at $\alpha_1=0^\circ$, $\alpha_2=45^\circ$, and $\alpha_3=90^\circ$. (1) Find ε_{xx}, ε_{yy} and ε_{xy} at this position. (2) Find the general expressions for the principal strains and maximum shear strains in terms of measurable values ε_{0°, ε_{45° and ε_{90°. (30 points) $\varepsilon_{xx_\alpha} = \varepsilon_{xx}\cos^2(\alpha) + 2\varepsilon_{xy}\cdot\cos(\alpha)\sin(\alpha) + \varepsilon_{yy}\sin^2(\alpha)$

1)

$\varepsilon_{0^\circ} = \varepsilon_{xx} \Rightarrow \varepsilon_{xx} = \varepsilon_0$

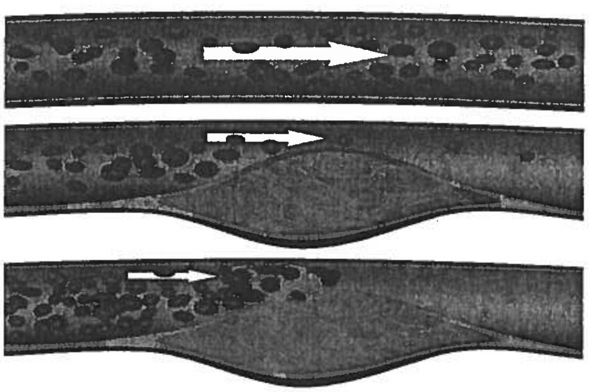
+15 $\quad \varepsilon_{45^\circ} = \frac{1}{2}\varepsilon_{xx} + 2\cdot\varepsilon_{xy}\cdot\frac{1}{2} + \frac{1}{2}\varepsilon_{yy} \Rightarrow \varepsilon_{xy} = \varepsilon_{45^\circ} - \frac{\varepsilon_0+\varepsilon_{90}}{2}$

$\varepsilon_{90^\circ} = \varepsilon_{yy} \Rightarrow \varepsilon_{yy} = \varepsilon_{90^\circ}$

2) $$\varepsilon_{1,2} = \frac{\varepsilon_{xx}+\varepsilon_{yy}}{2} \pm \sqrt{\left(\frac{\varepsilon_{xx}-\varepsilon_{yy}}{2}\right)^2 + \varepsilon_{xy}^2} \Rightarrow \varepsilon_{1,2} = \frac{\varepsilon_0+\varepsilon_{90}}{2} \pm \sqrt{\left(\frac{\varepsilon_0+\varepsilon_{90}}{2}\right)^2 + \left(\varepsilon_{45} - \frac{\varepsilon_0+\varepsilon_{90}}{2}\right)^2}$$

15 $$\varepsilon'_{xy,max} = \sqrt{\left(\frac{\varepsilon_{xx}-\varepsilon_{yy}}{2}\right)^2 + \varepsilon_{xy}^2} \Rightarrow \varepsilon'_{xy,max} = \sqrt{\left(\frac{\varepsilon_0-\varepsilon_{90}}{2}\right)^2 + \left(\varepsilon_{45} - \frac{\varepsilon_0+\varepsilon_{90}}{2}\right)^2}$$

4. Lab Question: Atherosclerosis is a vascular disease wherein lipoproteins and leukocytes are accumulated under endothelial cells, forming a plaque at lesion zone that might cause stroke if ruptures (see the image). Imagine a group of surgeons are discussing about a patient with an acute atherosclerosis lesion in one of their arteries. A surgeon suggests confining the outer surface of the vessel at the lesion zone with a bio-compatible, bandage-like device in order to prevent the vessel from further bulging that would otherwise lead to vessel rupture and internal bleeding. As a bioengineer, you are also asked to attend the meeting. Explain how you would evaluate this suggestion. (Hint: The main concern is if an external confinement around the vessel would reduce the risk of rupture. Remember your results from Lab3.) (10 points)



Healthy Vessel

Atherosclerosis

Plaque Rupture

Fixing the outer boundary will lead to a collapse of the inner lumen. Would not suggest this method. If the answer includes anything from COMSOL you got points, but that depended on the amount of detail in the model.

Appendix. Equations

The followings are the strain transformation equations. Stress transformation has the same set of equations (replacing strain with stress respectively).

$$\varepsilon'_{xx} = \varepsilon_{xx} \cos^2 \alpha + 2\varepsilon_{xy} \sin\alpha \cos\alpha + \varepsilon_{yy} \sin^2 \alpha,$$

$$\varepsilon'_{yy} = \varepsilon_{xx} \sin^2 \alpha - 2\varepsilon_{xy} \sin\alpha \cos\alpha + \varepsilon_{yy} \cos^2 \alpha,$$

$$\varepsilon'_{xy} = 2 \sin\alpha \cos\alpha \left(\frac{\varepsilon_{yy} - \varepsilon_{xx}}{2} \right) + (\cos^2 \alpha - \sin^2 \alpha)\varepsilon_{xy},$$

$$\varepsilon_{1,2} = \varepsilon'_{xx})_{\text{max/min}} = \varepsilon'_{yy})_{\text{max/min}} = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} \pm \sqrt{\left(\frac{\varepsilon_{xx} - \varepsilon_{yy}}{2} \right)^2 + \varepsilon_{xy}^2},$$

with

$$\alpha_p = \frac{1}{2} \tan^{-1}\left(\frac{\varepsilon_{xy}}{(\varepsilon_{xx} - \varepsilon_{yy})/2} \right).$$

$$\varepsilon'_{xy})_{\text{max/min}} = \varepsilon'_{xy}(\alpha_s) = \pm\sqrt{\left(\frac{\varepsilon_{xx} - \varepsilon_{yy}}{2} \right)^2 + \varepsilon_{xy}^2},$$

where

$$\alpha_s = \frac{1}{2} \tan^{-1}\left(\frac{\varepsilon_{yy} - \varepsilon_{xx}}{2\varepsilon_{xy}} \right).$$