

E 120: Principles of Engineering Economics

Practice Final Exam
Instructor: Ilan Adler

Name: _____ (please print)

SID: _____

- **Any communication with other students during the exam (including showing, viewing or sharing any writing) is strictly prohibited. Any violation will result in a score of 0 points for the exam.**
- Clearly state all the mathematical expressions that are needed to solve the problems. **No credit will be given to numerical answers without the proper setup.**
- Answer each of the following questions in the space provided. If you need more space to show major computations you performed to obtain your answer for a particular problem, use the back of the preceding page.
- You can quote and use any result stated in class or in the main body of the textbook as well as well-known general mathematical results but **no** references to other sources (including textbook exercises) are allowed.
- Present your work in an organized and neat fashion.
- Good luck!

1. Consider a bank loan for \$150,000 with an *adjustable* APR of 5%, compounded monthly. The loan is paid by a fixed monthly payment plan, where the first payment is due **the day the loan is taken** and there are a total of 240 payments.
 - (a) What is the monthly payment amount?
 - (b) After having paid the 60th installment, you are informed by the bank that the APR has risen to 6%.
 - i. What is the loan balance at that moment? (The time of reference is the 60th month)
 - ii. Suppose you keep the same monthly payments as before, how long will it take to pay off the loan?
Note: Non-integer solution is okay. And you should express your answer as the number of months from the 60th month.

2. Stocks A and B are both selling for \$100 right now. The risk-free rate is 2%. Suppose there are three possible cases of the market condition in one year from now, each with equal probability. The corresponding stock prices are as given:

Condition	Price of stock A	Price of stock B
Good	130	140
Average	105	93
Bad	80	85

- (a) Find the annual return rate of each stock in each case (i.e. good / average / bad). Then find the expected return and variance of each stock's return and the covariance of the two stock returns.
For next questions, we assume that there stock A and Stock B are the only assets on the market.
- (b) Find the market portfolio and its expected return.
- (c) If your desired return is 4%, how should you allocate your money on each of the asset (i.e., risk-free asset, stock A and stock B)?

3. Assume that the expected return rate of the market is 12%, while the risk-free rate is 4%. An investor in this market is going to create a portfolio which consists of stock A and stock B. The expected return rate of stock A is 20% more than stock B, while the β of stock A is 25% more than the β of stock B. Suppose the expected returns of both stocks lie on the Security Market Line (SML). Furthermore, suppose the variances for stock A and stock B are 0.36, and 0.25, respectively. The correlation between them is 0.6. Short selling is allowed in this market.
 - (a) What are the expected return rates for stock A and stock B?

- (b) What are the β 's of stock A and stock B?
 - (c) What is the minimum variance the investor can achieve by investing in stock A and stock B? What does this portfolio look like?
 - (d) If the investor wants an expected return rate of 30% by investing in stock A, stock B, and the risk-free asset, what are the minimum variance and the weights for each stock and the risk-free asset?
4. Suppose that there are only two stocks available in the market, A and B. The returns are expected at the end of the year. Stock A has variance of return at 0.10; stock B has variance of return at 0.15 and a β of 0.7. It is also given that the covariance between A and B is 0.06 and that the risk-free rate is 2%. Borrowing and depositing unlimited quantities of money are allowed at the risk-free rate. Your friend, the universally acknowledged market expert, knows the expected returns of each stock and uses the Markowitz model to construct an efficient portfolio for the expected return that she seeks. She plans to deposit \$600 in the bank at the risk-free rate, invest \$1200 in stock A, and invest \$1200 in stock B. She expects her investment to be worth \$3300 at the end of the year.

Suppose you have \$1000 to invest in any combination of A, B, and the risk free asset. You expect your investment to be worth \$1200 at the end of the year.

- (a) What is the expected return of the market portfolio? What is its variance?
 - (b) How much should you invest in stock A, stock B, and the risk-free asset, in order to minimize the variance of your portfolio? What is the variance of your portfolio?
 - (c) Assume CAPM holds, calculate the β of stock A.
 - (d) What is the covariance of stock A with the market portfolio? What is the covariance of the portfolio you constructed in part (b), with the market portfolio?
5. Consider a casino game in which a wheel with 11 spots numbered 0-10 is used. At every turn any of the 11 numbers can come up with equal probabilities. There are three bets available with the following payoffs for a bet of \$1 (where x is the outcome of the wheel).
- Bet 1:** If x turns out to be 0, you lose \$9. Otherwise you win \$1.
- Bet 2:** If x turns out to be 1-5, you lose \$1. Otherwise you win \$1.
- Bet 3:** If x turns out to be 6-10, you lose \$1. Otherwise you win \$1.
- (a) What are the expected values and variances of the three bets?
- For the next questions assume that you can bet \$1000 once.**
- (b) How would you allocate the \$1000 if you can bet only on Bet 1 and Bet 2 with the goal of having minimum variance?

- (c) Can you make a complete arbitrage (that is, guarantee a positive profit regardless of the outcome) by some allocation of the \$1000 among the three bets? Explain your answer.
6. Please indicate whether each of the following statements is true or false. If it is true, explain/prove why it is true. If it is false, explain why or provide a counterexample. Credit will not be given unless an explanation is provided.
- (a) Bond A is a 10-year, semi-annual coupon bond with face value \$1000 and coupon payment of \$50. Bond B is a 20-year, annual coupon bond with face value \$4000 and coupon payment of \$200. Given the yield to maturity of both bonds are the same, and are less or equal to 15%, the calculated price of bond A is less than the calculated price of bond B (assume that all payments are certain).
- (b) Your little sister proposes the following game to you. On day 1, you will give her \$1. On day 2, she will give you \$2. And in general, you will give her \$ i on day i if i is odd, and she will give you \$ i on day i if i is even (so that the cash flow from your point of view is $(0, -1, 2, -3, 4, -5, 6, \dots)$). If the interest rate per day is 10%, and you play the game for 100 days, then you should take up the offer.
- (c) If an asset is uncorrelated with the market portfolio, then the expected rate of return of this asset must be equal to the risk-free rate.
- (d) If the risk-free rate increases (everything else being equal), then the variance of the market portfolio decreases.