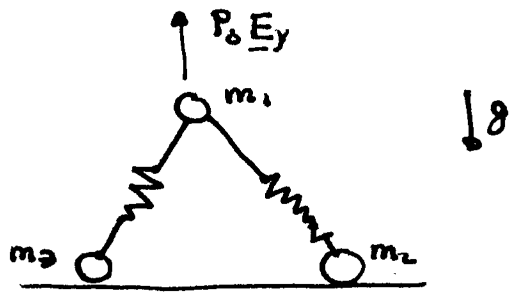
Problem 1



(a) $\underline{r}_1 = y\,\underline{E}_y \qquad \underline{r}_2 = -\underline{r}_3 = x\,\underline{E}_x \qquad m_2 = + m_3$

$x_2 = -x_3 = x$

$$\underline{r} = \frac{m_1}{m_1+m_2+m_3}\,y\,\underline{E}_y + 0\,\underline{E}_x = y_c\,\underline{E}_y$$

$$\underline{G} = m\dot{\underline{r}} = \frac{m_1(m_1+m_2+m_3)}{m_1+m_2+m_3}\,\dot{y}\,\underline{E}_y = m_1\dot{y}\,\underline{E}_y$$

$$\underline{H}_c = (\underline{r}_1-\underline{r})\times m_1\underline{v}_1 + (\underline{r}_2-\underline{r})\times m_2\underline{v}_2 + (\underline{r}_3-\underline{r})\times m_3\underline{v}_3$$

$$= (y-y_c)\underline{E}_y \times m_1\dot{y}\,\underline{E}_y + (x_2\underline{E}_x - y_c\underline{E}_y)\times m_2\dot{x}_2\underline{E}_x$$

$$+ (x_3\underline{E}_x - y_c\underline{E}_y)\times m_3\dot{x}_3\underline{E}_x$$

$$= (m_2\dot{x}_2 y_c + m_3\dot{x}_3 y_c)\,\underline{E}_z$$

$= \underline{0}$ because $m_2 = m_3$ and $\dot{x}_2 = -\dot{x}_3 = \dot{x}$

$$\underline{H}_o = \underline{H}_c + \underline{r}\times m\underline{v} \qquad m = m_1+m_2+m_3$$

$$= \underline{0} + y_c\underline{E}_y \times m\,\dot{y}_c\underline{E}_y$$

$$= \underline{0} + \underline{0} = \underline{0}$$

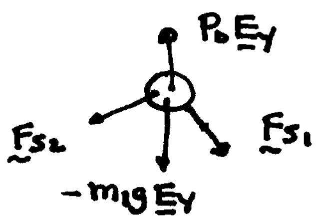
[Can also use definition $\underline{H}_o = \sum_{i=1}^{3} \underline{r}_i \times m_i\underline{v}_i$ to get same result]

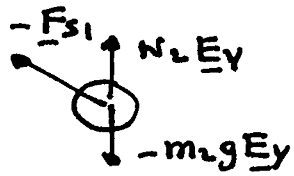
$$T = \tfrac{1}{2}m_1\underline{v}_1\cdot\underline{v}_1 + \tfrac{1}{2}m_2\underline{v}_2\cdot\underline{v}_2 + \tfrac{1}{2}m_3\underline{v}_3\cdot\underline{v}_3$$

$$= \tfrac{1}{2}m_1\dot{y}^2 + \tfrac{1}{2}m_2\dot{x}^2 + \tfrac{1}{2}m_3\dot{x}^2$$

$$\tfrac{1}{2}m\underline{v}\cdot\underline{v} = \tfrac{1}{2}m\dot{y}_c^2 = \frac{m_1^2}{2m}\dot{y}^2 \neq T$$

(b)





$$\underline{F}_{s_1} = -K(\|\underline{r}_1 - \underline{r}_2\| - \ell_0)\frac{\underline{r}_1 - \underline{r}_2}{\|\underline{r}_1 - \underline{r}_2\|}$$

$$\underline{r}_1 - \underline{r}_2 = y\underline{E}_y - x\underline{E}_x$$

$$\underline{F}_{s_2} = -K(\|\underline{r}_1 - \underline{r}_3\| - \ell_0)\frac{\underline{r}_1 - \underline{r}_3}{\|\underline{r}_1 - \underline{r}_3\|}$$

(c) From $(\underline{F}_1 = m_1\underline{a}_1)\cdot\underline{E}_y$

$$\begin{aligned} m_1\ddot{y} &= P_0 - m_1 g + (\underline{F}_{s_1} + \underline{F}_{s_2})\cdot\underline{E}_y \\ &= P_0 - m_1 g - K(\|\underline{r}_1 - \underline{r}_2\| - \ell_0)\frac{y}{\|\underline{r}_1 - \underline{r}_2\|} \\ &\qquad - K(\|\underline{r}_1 - \underline{r}_3\| - \ell_0)\frac{y}{\|\underline{r}_1 - \underline{r}_3\|} \\ &= P_0 - m_1 g - 2K(\|\underline{r}_1 - \underline{r}_2\| - \ell_0)\frac{y}{\|\underline{r}_1 - \underline{r}_2\|} \\ &= P_0 - m_1 g - 2K(\sqrt{x^2+y^2} - \ell_0)\frac{y}{\sqrt{x^2+y^2}} \end{aligned}$$

(d)

$$\begin{aligned} \dot{E} &= \underline{F}_{nc_1}\cdot\underline{v}_1 + \underline{F}_{nc_2}\cdot\underline{v}_2 + \underline{F}_{nc_3}\cdot\underline{v}_3 \\ &= \underline{0}\cdot\underline{v}_1 + N_2\underline{E}_y\cdot\underline{v}_2 + N_3\underline{E}_y\cdot\underline{v}_3 \\ &= 0 + 0 + 0 \end{aligned}$$

$\Rightarrow$ E is conserved.

Here $E = T + m_1 g y - P_0 y + K(\|\underline{r}_1 - \underline{r}_2\| - \ell_0)^2$

$P_0\underline{E}_y$ is conservative because it has a potential energy $-P_0\underline{E}_y\cdot\underline{r}_1 = -P_0 y$

where $-\frac{\partial}{\partial \underline{r}_1}\left(-P_0\underline{E}_y\cdot\underline{r}_1\right) = P_0\underline{E}_y$.

Note also that we used the identity

$$K(\|\underline{r}_1 - \underline{r}_2\| - \ell_0)^2 = K(\|\underline{r}_1 - \underline{r}_3\| - \ell_0)^2$$

in writing down E.

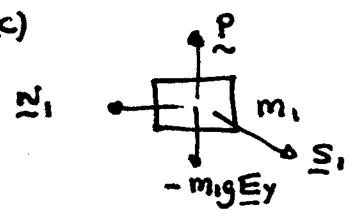
Problem 2

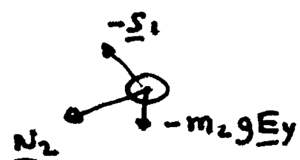
(a) $\dot{\underline{r}}_1 = \dot{y}E_y \qquad \dot{\underline{r}}_2 = \dot{y}\underline{E}_y + l\dot{\theta}(\underline{e}_\theta = \cos\theta\,\underline{E}_y - \sin\theta\,\underline{E}_x)$

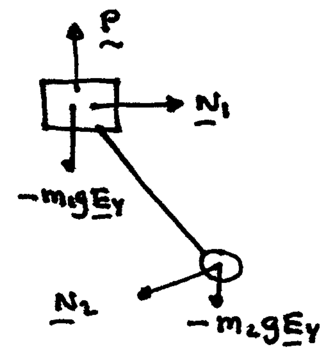
$$\underline{G} = m_1\dot{\underline{r}}_1 + m_2\dot{\underline{r}}_2 = (m_1+m_2)\dot{y}\,\underline{E}_y + m_2 l\dot{\theta}\,\underline{e}_\theta$$

(b)
$$\begin{aligned} T &= \tfrac{1}{2}m_1\underline{v}_1\cdot\underline{v}_1 + \tfrac{1}{2}m_2\underline{v}_2\cdot\underline{v}_2 \\ &= \tfrac{1}{2}m_1\dot{y}^2 + \tfrac{1}{2}m_2\dot{y}^2 + \tfrac{1}{2}m_2 l^2\dot{\theta}^2 + m_2 l\dot{\theta}\dot{y}\,\underline{E}_y\cdot\underline{e}_\theta \\ &= \tfrac{1}{2}(m_1+m_2)\dot{y}^2 + \tfrac{1}{2}m_2 l^2\dot{\theta}^2 + m_2 l\dot{\theta}\dot{y}\cos\theta \end{aligned}$$

(c)







$\underline{N}_1 = N_{1x}\underline{E}_x + N_{1z}\underline{E}_z \quad , \quad \underline{N}_2 = N_{2z}\underline{E}_z$

$S_1 = S_1\,\underline{e}_r$

(d) $E = m_1 g y + m_2 g(y + l\sin\theta) + T$

$$\begin{aligned} \dot{E} &= \underline{N}_1\cdot\underline{v}_1 + \underline{P}\cdot\underline{v}_1 + \underline{S}_1\cdot\underline{v}_1 - \underline{S}_1\cdot\underline{v}_2 + \underline{N}_2\cdot\underline{v}_2 \\ &= 0 + \underline{P}\cdot\underline{v}_1 + \underline{S}_1\cdot(l\dot{\theta}\,\underline{e}_\theta) + 0 \\ &= \underline{P}\cdot\underline{v}_1 + S_1\underline{e}_r\cdot l\dot{\theta}\underline{e}_\theta = \underline{P}\cdot\underline{v}_1 \end{aligned}$$

Hence $E(t_1) - E(t_0) = \int_{t_0}^{t_1}\underline{P}\cdot\underline{v}_1\,dt$ = Work done by $\underline{P}$

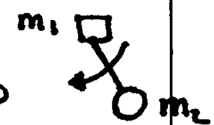
(e) As $\underline{P} = \underline{0}$ and $y = 0$, the expression for E simplifies to

$$E = \tfrac{1}{2} m_2 \ell^2 \dot{\theta}^2 + m_2 g \ell \sin\theta$$

Initially $\quad E = E_0 = m_2 g \ell \sin\theta_0$

Just before impact $\quad E = E_1 = \tfrac{1}{2} m_2 \ell^2 \dot{\theta}^2 - m_2 g \ell$

Hence $$(\ell\dot{\theta})^2 = +2g\ell(\sin\theta_0 + 1) \qquad (\dot{\theta} < 0$$



Impulse experienced by Pumpkin is equal and opposite to the impulse experienced by the sphere of mass m_2:

$$\int \underline{R}\, dt = + m_2 \ell \dot{\theta}\, \underline{E}_X$$

where $$\ell\dot{\theta} = -\sqrt{2g\ell(1 + \sin\theta_0)}$$

Hence $$\int \underline{R}\, dt = - m_2 \sqrt{2g\ell(1 + \sin\theta_0)}\, \underline{E}_X$$

$$= \text{Impulse on Pumpkin}$$