EE 120 SIGNALS AND SYSTEMS, Fall 2012

Midterm # 2, November 5, Monday, 10:10-11:50 am

Name SOLUTIONS

Closed book. Two letter-size cheatsheets are allowed. Show all your work. Credit will be given for partial answers.

Problem	Points	~~Score~~ Grader
1	20	YS
2	20	KS
3	20	MA
4	20	YS
5	20	KS
Total	100	

1. Given the frequency response:

$$H(j\omega) = 10\frac{1-j\omega}{1+j\omega}$$

a) (5 points) Sketch the Bode plot, labeling the relevant frequencies, and magnitude and phase values.

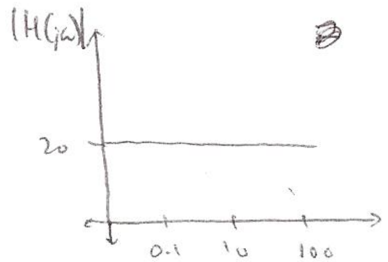
b) (5 points) Give an exact expression for the phase delay.

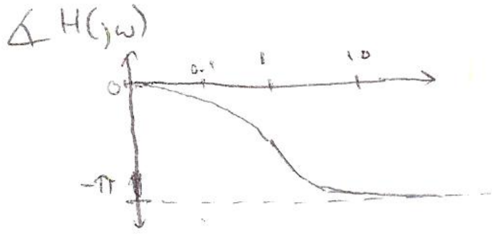
c) (5 points) Give an exact expression for the group delay.

d) (5 points) Determine the response of the system to the input $x(t) = \cos(t)$.

(a) $20 \log |H(j\omega)| = 20 + \log\left(\frac{|1-j\omega|}{|1+j\omega|}\right) = 20$

$\angle H(j\omega) = \angle\left(\frac{1}{1+j\omega}\right) + \angle(1-j\omega) = -2\tan^{-1}(\omega)$





(b) $\tau_p = -\frac{\angle H(j\omega)}{\omega} = \frac{2\tan^{-1}(\omega)}{\omega}$.

(c) $\tau_g = -\frac{d}{d\omega}\angle H(j\omega) = \frac{2}{1+\omega^2}$

(d) $x(t) = \frac{1}{2}\left(e^{jt} + e^{-jt}\right)$

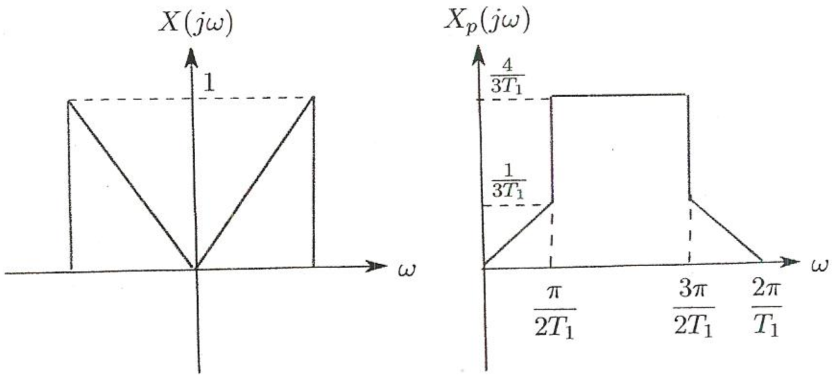
$y(t) = \frac{1}{2}H(j(1))e^{jt} + \frac{1}{2}H(j(-1))e^{-jt} = \frac{10}{2}\left(\frac{1-j}{1+j}e^{jt} + \frac{1+j}{1-j}e^{-jt}\right)$

$= \frac{10}{2\cdot 2j}\left(2e^{jt} - 2e^{-jt}\right) = 10\sin(t)$.

Alt.. $\tau_p(1) = 2\tan^{-1}(1) = \frac{\pi}{2}$. $y(t) = |H(j1)|\cos(t - \tau_p \cdot 1) = 10\cos(t - \frac{\pi}{2})$

$= 10\sin(t)$.

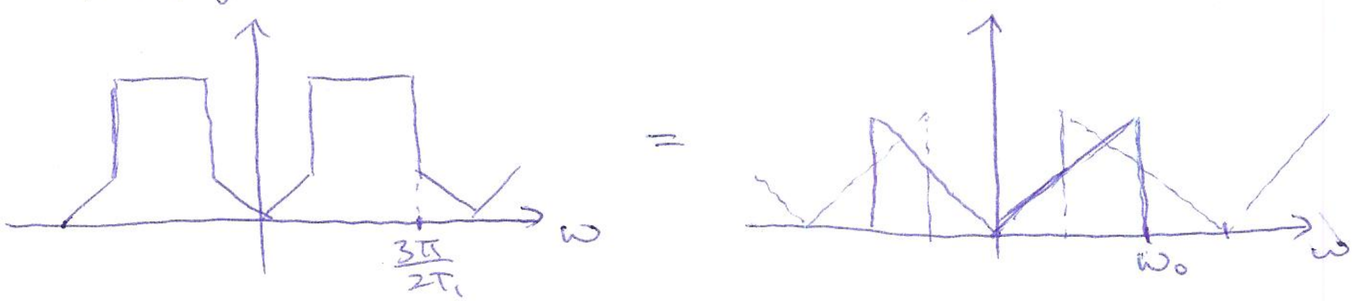
2. A continuous time signal $x(t)$ with the spectrum $X(j\omega)$ depicted below is impulse-train sampled with period T_1. One period of the resulting spectrum $X_p(j\omega)$ is shown below.



a) (10 points) Determine the largest sampling period T_2 that would avoid aliasing, and express it in terms of T_1.

b) (10 points) Sketch the spectrum $X_p(j\omega)$ for each of the following two sampling periods: (i) $T = T_2$, (ii) $T = 2T_2$.

a) $X_p(j\omega)$ results from aliasing:



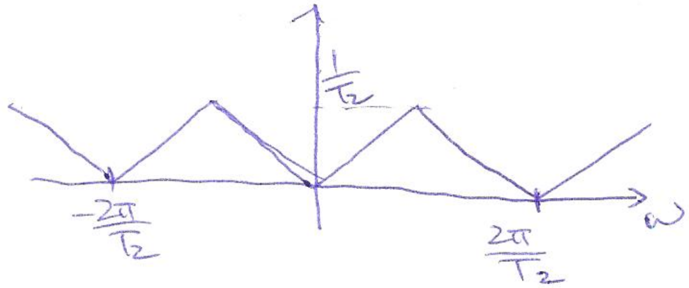
We see by inspection that $X(j\omega)$ is bandlimited to

$\left[-\frac{3\pi}{2T_1}, \frac{3\pi}{2T_1}\right]$

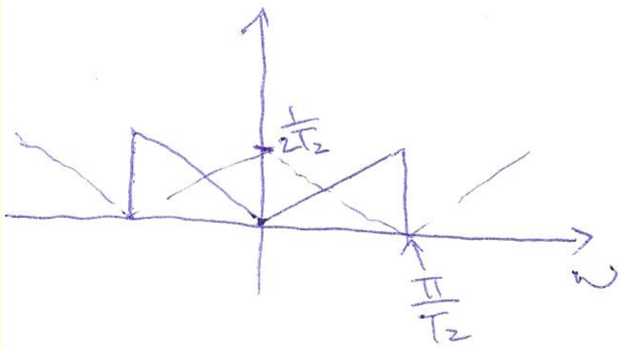
$\therefore$ Choose $\frac{2\pi}{T_2} = 2 \times \frac{3\pi}{2T_1} \Rightarrow T_2 = \frac{2}{3}T_1$

Additional workspace for Problem 2

b) $T = T_2$ $\rightarrow$ no aliasing



$T = 2T_2$ $\rightarrow$ copies at $\frac{\pi}{T_2}$



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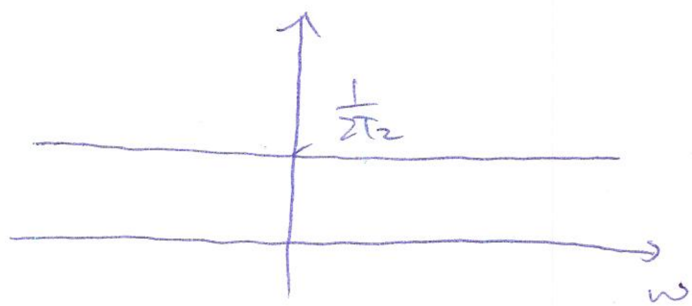

3. The continuous-time signal:

$$x(t) = \cos(\omega_0 t)$$

is sampled with frequency $\omega_s = \frac{3\omega_0}{2}$.

a) (5 points) Determine the resulting discrete-time signal $x[n]$.

b) (8 points) Suppose $x[n]$ is *upsampled* with $N = 2$ (that is, expanded in time by a factor of $N = 2$ and the missing samples are obtained from the existing ones with bandlimited interpolation). What is the resulting signal $x_u[n]$?

c) (7 points) Suppose now $x(t)$ is sampled with frequency $2\omega_s$. Do the samples match $x_u[n]$ in part (b)? For which values of ω_s would sampling followed by upsampling with N generate the same result as sampling with rate $N\omega_s$?

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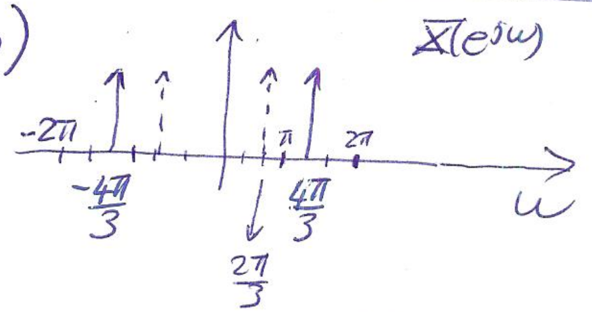
a) $T = \frac{2\pi}{\omega_s} = \frac{4\pi}{3\omega_0}$

$x[n] = x(nT) = \cos\left(\omega_0 n \frac{4\pi}{3\omega_0}\right) = \cos\left(\frac{4\pi}{3}n\right)$ ($= T$)

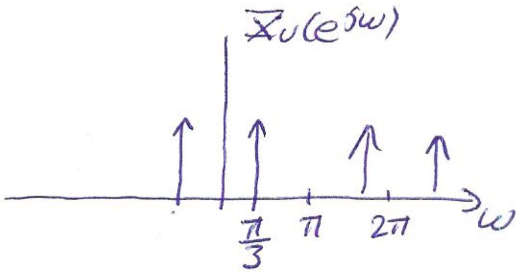
$= \cos\left(\frac{2\pi}{3}n\right)$

b)



expansion

bandlimited interpolation (i.e., LPF with cutoff π/N)



$x_u[n] = \cos\left(\frac{\pi}{3}n\right)$ (See also lecture 13, p.4, for a time domain version of this for the same example.)

c) $x_{new}[n] = x\left(n\frac{T}{2}\right) = \cos\left(\frac{2\pi}{3}n\right)$.

Therefore, $x_u[n] \neq x_{new}[n]$. They match if there is no aliasing in the first place, i.e., $\omega_s > 2\omega_0$.

(See Lecture 13)

4. a) (15 points) Find the right-sided signal $x(t)$ whose Laplace transform is:

$$X(s) = e^{-s}\frac{(s+1)^2}{s^2 - s + 1}$$

b) (5 points) Find the unilateral Laplace transform for $x(t)$ in part (a).

(a) Consider the term $\frac{(s+1)^2}{s^2-s+1}$. Rewrite using long division:

$$\begin{array}{r} 1 \\ s^2-s+1 \overline{\big) s^2+2s+1} \\ -(s^2-s+1) \\ \hline 3s \end{array} \Rightarrow = 1 + \frac{3s}{s^2-s+1}.$$

Write partial fraction expansion: $\frac{A}{s-(\frac{1}{2}+\frac{j\sqrt{3}}{2})} + \frac{B}{s-(\frac{1}{2}-\frac{j\sqrt{3}}{2})}$

$$\left.\begin{array}{l} s^2 - s + \frac{1}{4} = -\frac{3}{4} \\ (s-\frac{1}{2})^2 = -\frac{3}{4} \end{array}\right\} \Rightarrow s = \frac{1}{2} \pm \frac{j\sqrt{3}}{2}$$

So, $A(s-\frac{1}{2}+j\frac{\sqrt{3}}{2}) + B(s-\frac{1}{2}-j\frac{\sqrt{3}}{2}) = 3s.$

$\Rightarrow (A+B)s = 3s$

$A = 3-B.$

$\Rightarrow A(-\frac{1}{2}+j\frac{\sqrt{3}}{2}) + B(-\frac{1}{2}-j\frac{\sqrt{3}}{2}) = 0$

$(3-B)(-\frac{1}{2}+j\frac{\sqrt{3}}{2}) + B(-\frac{1}{2}-j\frac{\sqrt{3}}{2}) = 0$

$3(-\frac{1}{2}+j\frac{\sqrt{3}}{2}) - Bj\frac{\sqrt{3}}{2} - Bj\frac{\sqrt{3}}{2} = 0$

$\Rightarrow B = \frac{3}{2} + j\frac{\sqrt{3}}{2}, \; A = \frac{3}{2} - \frac{j\sqrt{3}}{2}.$

We get:

$$\delta(t) + \left[\left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right)e^{\left(\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)t} + \left(\frac{3}{2}+j\frac{\sqrt{3}}{2}\right)e^{\left(\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)t}\right]u(t)$$

$$= \delta(t) + \left[e^{\frac{t}{2}}\left[3\cos\left(\frac{\sqrt{3}}{2}t\right) + \sqrt{3}\sin\left(\frac{\sqrt{3}}{2}t\right)\right]\right]u(t)$$

using fact that $x(t-t_0) \xleftrightarrow{\mathcal{L}} e^{-st_0}X(s)$, we have

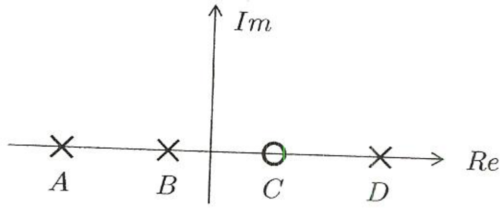
$$x(t) = \delta(t-1) + \left(e^{\frac{t-1}{2}}\right)\left(3\cos\left(\frac{\sqrt{3}}{2}(t-1)\right) + \sqrt{3}\sin\left(\frac{\sqrt{3}}{2}(t-1)\right)\right)u(t-1).$$

Additional workspace for Problem 4.

(b) Because $x(t)$ is (strictly) causal, its unilateral and bilateral Laplace transforms are equal. Thus

$$X(s) = e^{-s} \frac{(s+1)^2}{s^2 - s + 1}.$$

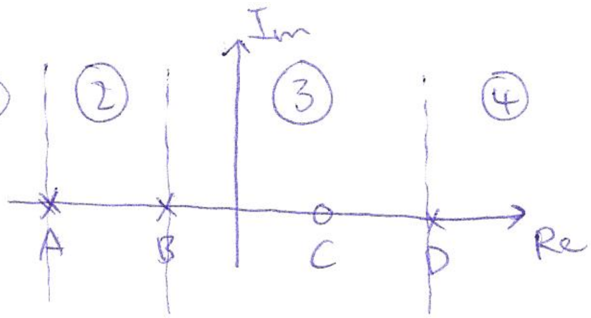
5. A LTI system has a rational transfer function with the pole/zero pattern below:



a) (10 points) Indicate all possible regions of convergence (ROC) that can be associated with this pattern.

b) (10 points) For each ROC identified in part (a), determine whether the system is stable and/or causal.

a)



1 $\mathrm{Re}(s) < A$

2 $A < \mathrm{Re}(s) < B$

3 $B < \mathrm{Re}(s) < D$

4 $\mathrm{Re}(s) > D$

b)

	Causal?	Stable?
1	✗	✗
2	✗	✗
3	✗	✓
4	✓	✗