Second Midterm Examination
Wednesday October 31 2012
Closed Books and Closed Notes

Question 1 *Planar Motion of a System of Three Particles* *(25 Points)*

As shown in Figure 1, a particle of mass m_1 is attached to a particle of mass m_2 by a linear spring of stiffness K and unstretched length ℓ_0 and a particle of mass $m_3 = m_2$ by a linear spring of stiffness K and unstretched length ℓ_0. The particles of mass m_2 and m_3 are free to move on a smooth horizontal surface and a constant force $\mathbf{P} = P_0\mathbf{E}_y$ acts on the particle of mass m_1.

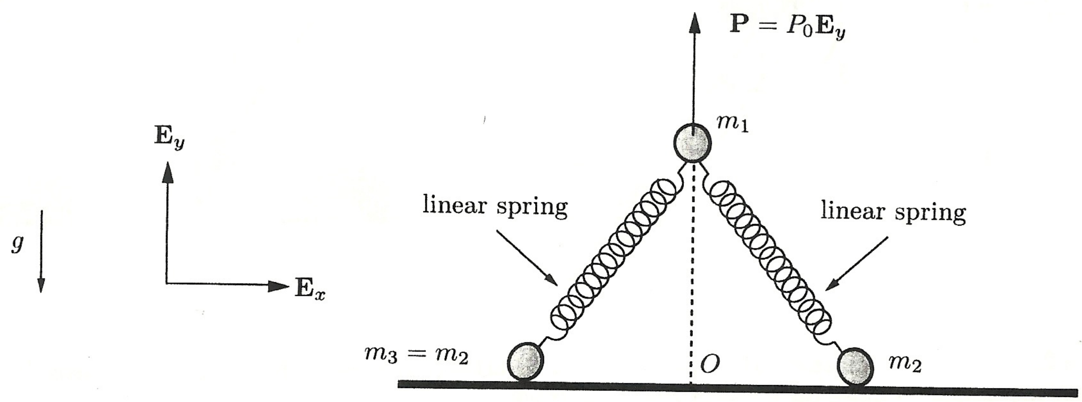

Figure 1: A system of three particles, two of are free to move on a smooth horizontal surface.

Relative to a fixed origin O, in this problem the position vectors of the particles have the representations

$$\mathbf{r}_1 = y\mathbf{E}_y, \qquad \mathbf{r}_2 = x\mathbf{E}_x, \qquad \mathbf{r}_3 = -x\mathbf{E}_x. \tag{1}$$

(a) *(10 Points)* Establish expressions for the position vector $\bar{\mathbf{r}}$ of the center of mass of the system, the linear momentum $\mathbf{G}$ of the system, the angular momenta $\mathbf{H}_C$ and $\mathbf{H}_O$, and the kinetic energy T of the system. In your solution, show that 2 of these 5 kinematic quantities are $\mathbf{0}$. In addition, show that the kinetic energy of the system of particles is not equal to the kinetic energy of the center of mass.

(b) *(5 Points)* Draw free body diagrams of the particles of mass m_1 and m_2. In your solution, give a clear expression for the spring forces acting on these particles.

(c) *(5 Points)* Show that the equation governing the motion of m_1 is

$$m_1\ddot{y} = P_0 + ???? \tag{2}$$

For full credit, supply the two missing terms.

(d) *(5 Points)* With the help of the work-energy theorem for a system of particles, prove that the total energy E of the system is conserved. In your solution, give a clear expression for E and verify that $\mathbf{P}$ is conservative.

Question 2 *Smashing Pumpkins* *(25 Points)*

In an effort to improve the overall Halloween experience for adults, an entrepreneur proposes the design of a pumpkin smasher which is loosely based on an impact testing machine. One of the key design problems facing the entrepreneur is to determine the size of the pumpkin smasher. As shown in Figure 2(a), the simplest model of the pumpkin smasher is a rod of length ℓ at the end of which is suspended a particle of mass m_2. The other end of the rod is attached by a pin-joint to a particle of mass m_1. An applied force $\mathbf{P}$ moves m_1 on a smooth vertical rail. The resulting motion of m_1 imparts a velocity to m_2 and, in the ensuing motion of m_2, a collision featuring m_2 and the pumpkin becomes eminent.

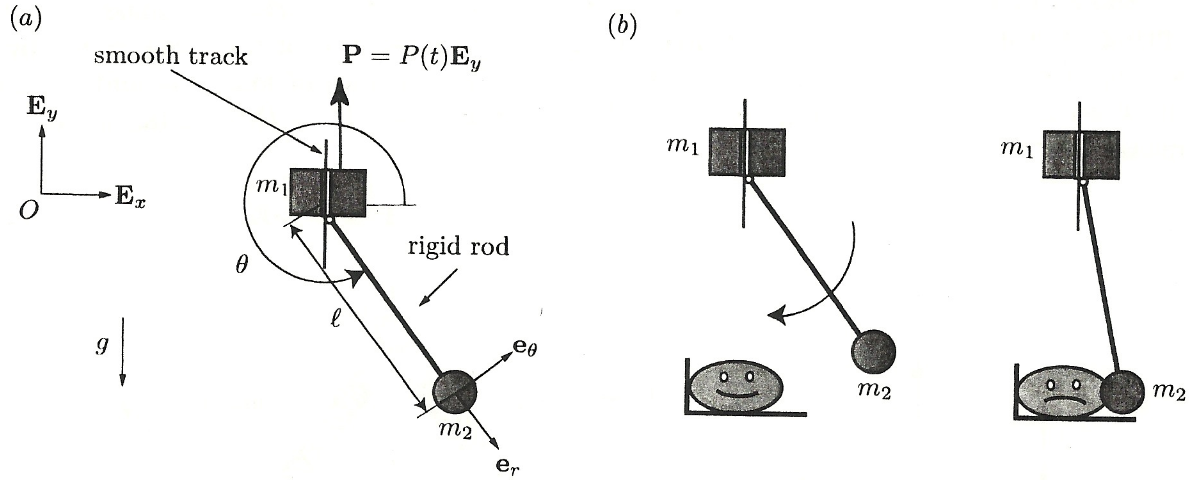

Figure 2: A particle of mass m_2 is suspended from a particle of mass m_1 by a rigid rod of length ℓ. The particle of mass m_1 is free to move on a smooth vertical track.

(a) *(4 Points)* Starting from the representations for the position vectors of m_1 and m_2,

$$\mathbf{r}_1 = y\mathbf{E}_y, \qquad \mathbf{r}_2 = \mathbf{r}_1 + \ell\mathbf{e}_r, \tag{3}$$

and assuming that m_1 and m_2 are in motion, establish an expression for the linear momentum $\mathbf{G}$ of the system.

(b) *(6 Points)* Show that the kinetic energy of the system has the representation

$$T = \frac{m_2}{2}\left(\ell^2\dot{\theta}^2\right) + ????. \tag{4}$$

For full credit supply the pair of missing terms.

(c) *(6 Points)* Draw 3 free-body diagrams: one for each of the individual particles and one for the system of particles. In your solution, give a clear expression for the tension force in the rod.

(d) *(4 Points)* Give an expression for the total energy E of the system of particles. Then, starting from the work-energy theorem for a system of particles,

$$\dot{E} = \mathbf{F}_{nc_1} \cdot \mathbf{v}_1 + \mathbf{F}_{nc_2} \cdot \mathbf{v}_2, \tag{5}$$

show that the change in E is due solely to the work performed by the applied force $\mathbf{P}$.

(d) *(5 Points)* Consider the simple case where m_1 is held at rest. As shown in shown in Figure 2(b), suppose that m_2 is released from rest with $\theta = \theta_0$. When $\theta = -\frac{\pi}{2}$, m_2 impacts a pumpkin which is held stationary. Assuming that the pumpkin and m_2 are both at rest immediately following the collision, show that the impulse exerted on the pumpkin during the collision is linearly dependent on m_2 and $\sqrt{\ell}$.