EECS 20N: Structure and Interpretation of Signals and Systems MIDTERM 1
Department of Electrical Engineering and Computer Sciences 18 September 2012
University of California Berkeley

LAST Name ______________________ FIRST Name ______________________

Lab Time ______________________

- **(10 Points)** Print your name and lab time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 70 minutes to complete. You will be given at least 70 minutes, up to a maximum of 80 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except one double-sided 8.5" $\times$ 11" sheet of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 8.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the eight numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

You may or may not find the following information useful:

$$
\begin{aligned}
e^{i\theta} &= \cos(\theta) + i\sin(\theta) \quad (\theta \in \mathbb{R}) \\
\cos(\theta) &= \frac{e^{i\theta} + e^{-i\theta}}{2} \quad (\theta \in \mathbb{R}) \\
\sin(\theta) &= \frac{e^{i\theta} - e^{-i\theta}}{2i} \quad (\theta \in \mathbb{R}) \\
\cos(\alpha + \beta) &= \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta. \\
\sin(\alpha + \beta) &= \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta. \\
\cos\alpha + \cos\beta &= 2\cos\left(\frac{\alpha+\beta}{2}\right) \cdot \cos\left(\frac{\alpha-\beta}{2}\right). \\
\sin\alpha + \sin\beta &= 2\sin\left(\frac{\alpha+\beta}{2}\right) \cdot \cos\left(\frac{\alpha-\beta}{2}\right). \\
\cos\left(\frac{\pi}{4}\right) &= \frac{1}{\sqrt{2}} \\
\sin\left(\frac{\pi}{4}\right) &= \frac{1}{\sqrt{2}}
\end{aligned}
$$

MT1.1 (35 Points) In this problem, z, z_1, and z_2 represent complex variables, whereas θ denotes a real variable. To avoid getting yourself into pathological arguments, do bear in mind that *neither* $\mathbb{C}$ *nor* $\mathbb{R}$ includes infinity.

(a) (10 Points) Prove that e^z is *nonzero* for every z.

(b) (10 Points) Prove that $\left|e^{i\theta}\right| = 1$ for every real θ.

(c) (10 Points) Prove that $e^z = 1$ if, and only if, $z = i2\pi k$, where k is an integer.

(d) (5 Points) True or False?

$$\text{If } e^{z_1} = e^{z_2}, \text{ then } z_1 = z_2.$$

Be sure to explain your reasoning. If you claim that the assertion is true, then you must prove it. If you claim that it's false, then you must give a counterexample; that is, find a pair of *distinct* complex numbers z_1 and z_2 for which the premise $e^{z_1} = e^{z_2}$ holds.

MT1.2 (30 Points)

(a) (15 Points) Show that for real-valued θ and ϕ,

$$e^{i\theta} + e^{i\phi} = A(\theta, \phi)\, e^{i\,B(\theta, \phi)},$$

where A and B are of the forms

$$A(\theta, \phi) = \alpha \cos\left[\beta \cdot (\theta - \phi)\right] \qquad \text{and} \qquad B(\theta, \phi) = \beta \cdot (\theta + \phi).$$

You must determine the *positive* real constants α and β.

(b) (15 Points) Let θ and ϕ denote functions of time, such that $\theta(t) = \pi t$ and $\phi(t) = \pi t/2$ for all t. Provide a well-labeled plot, over at least one complete cycle, for each of $|x(t)|$ and $\angle x(t)$, where $x(t) = e^{i\theta(t)} + e^{i\phi(t)}$.
If you're unsure of the values of α and β that you computed in part (a), you can still receive full credit in this part, if you label your plots in terms of α and β.

MT1.3 (40 Points) Consider the polynomial

$$\widehat{A}(z) = a_2\, z^2 + a_1\, z + a_0,$$

where $a_2 = 1$, $a_1 = -1/\sqrt{2}$, and $a_0 = 1/4$.

Consider a second polynomial

$$\widehat{B}(z) = b_2\, z^2 + b_1\, z + b_0$$

whose coefficients are given by $b_m = a_{2-m}$ for $m = 0, 1, 2$.

(a) (15 Points) Determine the roots of $\widehat{A}(z)$. Next, determine the roots of $\widehat{B}(z)$. Be sure to explain how the roots of $\widehat{B}(z)$ relate to those of $\widehat{A}(z)$.

(b) (10 Points) We construct the rational function $\widehat{H}(z)$ as follows:

$$\widehat{H}(z) = \frac{\widehat{B}(z)}{\widehat{A}(z)}.$$

Define the roots of $\widehat{B}(z)$ as the *zeros* of $\widehat{H}(z)$, and the roots of $\widehat{A}(z)$ as the *poles* of $\widehat{H}(z)$. Determine, and provide a well-labeled plot of, the zeros and poles of $\widehat{H}(z)$ on the complex plane. Identify each zero with the symbol "$\circ$" and each pole with the symbol $\times$. Describe how each root (pole or zero) is situated relative to the unit circle as well as to the other roots. Be sure to draw the unit circle in your diagram.

(c) (15 Points) Define the function $H(\omega)$ as follows:

$$H(\omega) = \widehat{H}(z)\Big|_{z=e^{i\omega}},$$

where ω is real. That is, $H(\omega)$ is obtained by evaluating $\widehat{H}(z)$ on the unit circle. Determine a reasonably simple expression for $|H(\omega)|$ for all ω.

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Problem	Points	Your Score
Name	10	
1	35	
2	30	
3	40	
Total	**115**	