EECS 20N: Structure and Interpretation of Signals and Systems MIDTERM 2
Department of Electrical Engineering and Computer Sciences 01 November 2012
University of California Berkeley

LAST Name ______________________ FIRST Name ______________________

Lab Time ______________________

- **(10 Points)** Print your name and lab time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 70 minutes to complete. You will be given at least 70 minutes, up to a maximum of 80 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except two double-sided 8.5" $\times$ 11" sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 8.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the eight numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

MT2.1 (30 Points) The impulse response of a system H is

$$\forall n \in \mathbb{Z}, \quad h(n) = -\alpha^n u(-n-1).$$

(a) (8 Points) Provide a well-labeled plot of $h(n)$ for each of the following cases:

(i) $\alpha = 2$

(ii) $\alpha = -1/2$

(b) (4 Points) Could the system H be memoryless? Explain succinctly, but clearly and convincingly.

(c) (8 Points) Express $y(n)$, the output of the system at sample n, in terms of the input signal x and the parameter α. Using your expression for $y(n)$—or by resorting to another succinct, but clear and convincing, reasoning—determine whether the system H is causal, anti-causal, or non-causal (neither causal nor anti-causal).

(d) (4 Points) Determine the values of α for which the system H is BIBO stable. Do note that α may be complex-valued. Explain succinctly, but clearly and convincingly.

(e) (6 Points) For those values of α that make the system H BIBO stable, determine a simple closed-form expression for $H(\omega)$, the frequency response of the system.

MT2.2 (30 Points) Consider a discrete-time FIR filter B whose impulse response b is real-valued. Suppose $b(n) = 0$ for n outside the interval $[0, N-1]$, where $N \in \{2, 3, 4, \ldots\}$. Let $B(\omega)$ denote the frequency response of this filter.

The impulse response of a related discrete-time filter A is described by $a(n) = b(-n)$ for all integer n.

(a) (10 Points) Show that $A(\omega) = B(-\omega)$ for all real ω, where $A(\omega)$ is the frequency response of the filter A.

(b) (20 Points) The frequency response of a discrete-time LTI filter H is given by $H(\omega) = B(\omega)/A(\omega)$. Determine the simplest expression for $|H(\omega)|$ and the simplest expression for $\angle H(\omega)$.

Next, suppose $b(n) = \frac{1}{2}[\delta(n) + \delta(n-1)]$. Determine a simple expression, and provide a well-labeled plot, for each of $|H(\omega)|$ and $\angle H(\omega)$.

MT2.3 (45 Points) Let f denote the impulse response of a causal, BIBO stable, LTI filter F. The impulse response g of a related filter G is the L-fold upsampled version of f. That is, for $L \in \{2, 3, 4, \ldots\}$, and for all integer n,

$$g(n) = \begin{cases} f\left(\frac{n}{L}\right) & \text{if } n \bmod L = 0 \\ 0 & \text{otherwise.} \end{cases}$$

Let $F(\omega)$ and $G(\omega)$ denote the frequency responses of the filters F and G, respectively.

(a) (10 Points) Show that $G(\omega) = F(L\omega)$.

(b) (10 Points) Suppose the input-output behavior of the filter F is described by the linear, constant-coefficient difference equation (LCCDE)

$$\sum_{k=0}^{N} a_k \, y(n-k) = \sum_{m=0}^{M} b_m \, x(n-m),$$

where x denotes the input and y the corresponding output of the filter F. Determine the LCCDE satisfied by the input x and corresponding output y of the filter G. Explain your reasoning succinctly, but clearly and convincingly.

(c) (25 Points) The frequency response of the filter F is given by

$$\forall \omega \in \mathbb{R}, \quad F(\omega) = \frac{1 - e^{-i\omega}}{1 - 0.99\, e^{-i\omega}}.$$

(i) (7 Points) Provide a delay-adder-gain block diagram implementation of the filter F.

(ii) (7 Points) Determine a reasonably simple expression for $f(n)$.

(iii) (5 Points) Provide a well-labeled plot of the magnitude response $|F(\omega)|$.

(iv) (6 Points) Suppose $L = 4$ and the input x to the filter G is

$$\forall n, \quad x(n) = \frac{1}{2} + \cos\left(\frac{\pi n}{2}\right) + \frac{1}{2}(-1)^n.$$

Determine a simple expression for the output y of the filter G.

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Problem	Points	Your Score
Name	10	
1	30	
2	30	
3	45	
Total	**115**	