UNIVERSITY OF CALIFORNIA AT BERKELEY
Department of Mechanical Engineering
ME132 Dynamic Systems and Feedback

Midterm I **Fall 2012**

Closed Book and Closed Notes. One 8.5 × 11 sheet (double-sided) of handwritten notes allowed. Scientific calculator with no graphics allowed.

Your Name: TAREK RABBANI

Please answer all questions. This exam has 11 pages.

Problem:	1	2	3	4	5	Total
Max. Grade:	20	15	40	10	15	100
Grade:						

1. Consider the following input-output differential equation:

$$\ddot{y}(t) + \dot{y}(t)u(t) + 2y(t) = 0$$

where $u(t)$ is the input and $y(t)$ is the output. The initial conditions are $\dot{y}(0) = 2$ and $y(0) = 1$.

(a) Is this system,

i. Linear? No, the term $\dot{y}(t)u(t)$ make the system non-linear.

ii. Time-invariant? Yes, let $y(t)$ be the output for an input $u(t)$ is $y(t-\tau)$ the output of an input $u(t-\tau)$? Is $\ddot{y}(t-\tau) + \dot{y}(t-\tau)u(t-\tau) + 2y(t-\tau) \overset{?}{=} 0$

$\ddot{y}(t') + \dot{y}(t')u(t') + 2y(t') \overset{?}{=} 0$

True! since t' is a dummy variable and $d(t-\tau) = dt'$

iii. Memoryless?

No, the terms $\ddot{y}$, and $\dot{y}u$ make the system dynamic

(b) Write down the differential equation in state-space form.

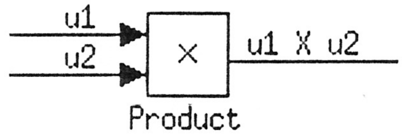
let $x_1 = y$ and $x_2 = \dot{y}$

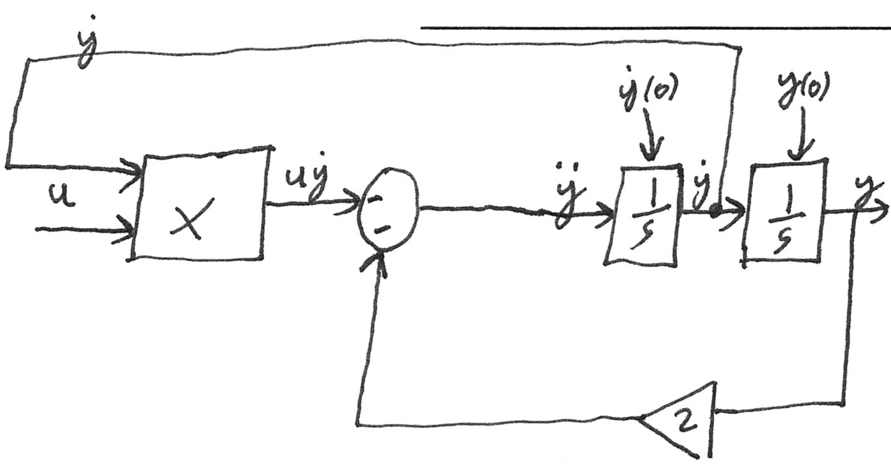
$\dot{x}_1 = x_2$

$\dot{x}_2 = \ddot{y} = -\dot{y}(t)u(t) - 2y(t)$

$$\Rightarrow \boxed{\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -2x_1 - x_2 u \\ x_1(0) &= 1 \\ x_2(0) &= 2 \end{aligned}}$$

(c) Sketch the Simulink block diagram for the system by composing integrators. Show also where the initial conditions $\dot{y}(0)$ and $y(0)$ are set.
Hint: To multiply two signal u_1 and u_2 in simulink, you can use the product block diagram as shown below:





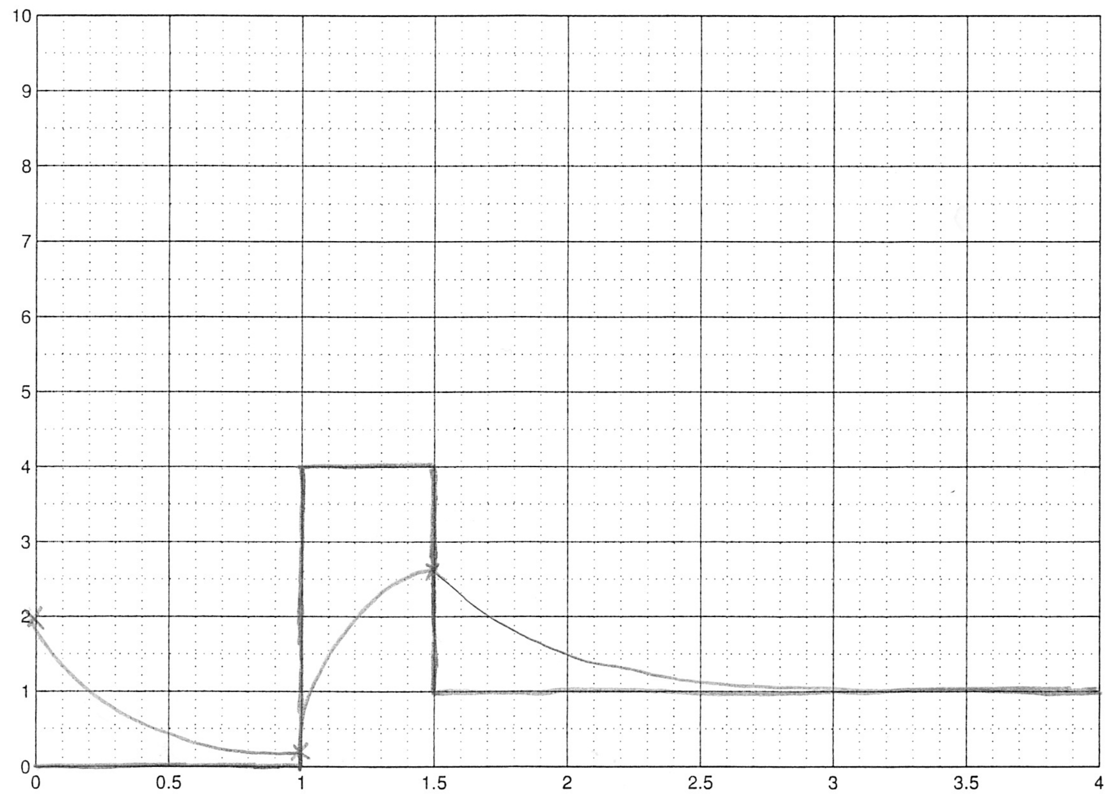
2. Consider the system described by the differential equation

$$\dot{y} + 2y = 2u + d$$

where $u(t)$ is the input, $d(t)$ is the disturbance, and $y(t)$ is the output. The initial condition is $y(0) = 2$.

Let $u(t) = \begin{cases} 0 & t < 1 \\ 4 & t \geq 1 \end{cases}$, and $d(t) = \begin{cases} 0 & t < 1.5 \\ -6 & t \geq 1.5 \end{cases}$.

Sketch the response $y(t)$ on the graph provided next.



$0 \leq t < 1$

$u = 0$, $d = 0$

$y(t) = y(0)\, e^{-2t}$

$= 2e^{-2t}$

$y(0) = 2$

$y(1) = 0.27$

$1 \leq t < 1.5$

$u = 4$, $d = 0$

$y_{ss} = \frac{2}{2} u_{ss}$

$= 4$

$t_0 = 1$

$y(t) = y_{ss} + e^{-2(t - t_0)} [y(t_0) - y_{ss}]$

$= 4 + e^{-2(t-1)} [y(1) - 4]$

$y(1.5) = 4 + e^{-2(0.5)} [0.27 - 4]$

$= 2.63$

$1.5 \leq t$

$u = 4$, $d = -6$

$y_{ss} = \frac{2}{2} u_{ss} + \frac{1}{2} d_{ss}$

$= 4 + \frac{1}{2}(-6)$

$= 1$

Time constant $= \frac{1}{2}$ s.

converges $\approx 3T = 1.5$s.

$y(3) \approx y_{ss} = 1$

3. A system is described by the following differential equation:

$$\dot{y}(t) - 2y(t) = u(t)$$

where $u(t)$ is the input and $y(t)$ is the output. The initial condition is $y(0) = 0$

(a) Is the system described by the SLODE stable?

$-2 < 0$ and the system is not stable

(b) We wish to control the SLODE using a controller of the form

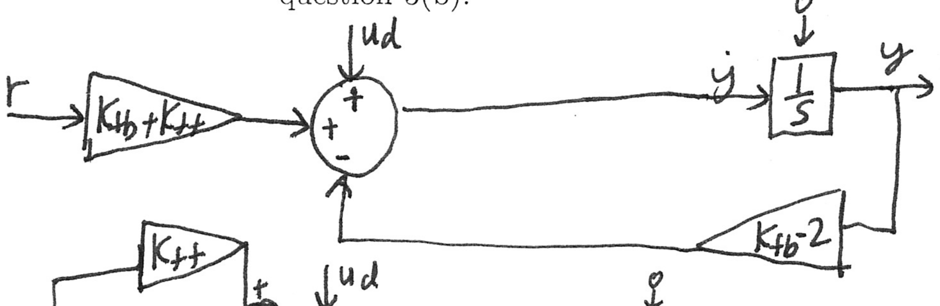
$$u(t) = K_{fb}(r - y) + K_{ff}r + u_d(t),$$

with K_{fb} the feedback gain , K_{ff} the feedforward gain, and $u_d(t)$ an input disturbance signal (an external signal we do not have control over). Write the closed-loop differential equation.

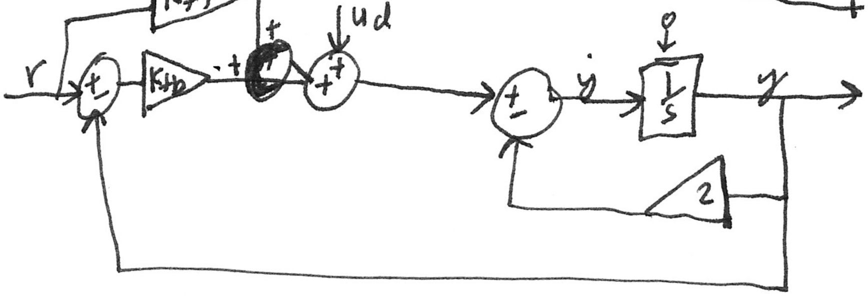
$$\dot{y} - 2y = K_{fb}(r-y) + K_{ff}r + u_d$$

$$\boxed{\dot{y} + (K_{fb} - 2)y = (K_{fb} + K_{ff})r + u_d}$$

(c) Sketch the Simulink block diagram for the closed-loop system in the previous question 3(b).



or



(d) Which values of the controller K_{fb} and K_{ff} guarantee closed-loop stability?

$K_{fb} - 2 > 0$

$\boxed{K_{fb} > 2}$

$\boxed{K_{ff} \in R}$ grades were not deducted if no statement is made on K_{ff}.

(e) Assume $K_{ff} = 0$. Can you design a controller K_{fb} such that

(1) the closed-loop system is stable,

(2) the steady-state output tracking error less than 10% when the reference signal is constant and when $u_d(t) = 0$, i.e. $|y_{ss} - r| < 0.1\,|r|$ with y_{ss} the steady-state output when $u_d(t) = 0$,

(3) The steady-state attenuation of a constant input disturbance is greater or equal than 90%, i.e.

$$\left|\frac{y_{ss}}{u_d}\right| < 0.1$$

with y_{ss} the steady-state output when $r = 0$ and u_d is constant.

(4) K_{fb} can not have a magnitude more than 20, i.e. $|K_{fb}| < 20$.

Report the value of the controller K_{fb} **if** you were able to find one. Otherwise claim that the control design problem has no solution.

(1) $K_{fb} > 2$

(2) $y_{ss} = \dfrac{K_{fb}}{K_{fb}-2}\, r$

$$\Rightarrow |y_{ss} - r| = \left|\left(\frac{K_{fb}}{K_{fb}-2} - 1\right) r\right| = \left|\frac{2}{K_{fb}-2}\, r\right| = \frac{2}{K_{fb}-2}\,|r|$$

Thus $|y_{ss} - r| < 0.1\,|r|$ is equivalent to

$$\frac{2}{K_{fb}-2} < 0.1 \Rightarrow 0.1\,K_{fb} - 0.2 > 2$$

$$\Rightarrow \boxed{K_{fb} > 22}$$

(3) In this case $y_{ss} = \dfrac{u_d}{K_{fb}-2}$

$$\left|\frac{y_{ss}}{u_d}\right| = \left|\frac{1}{K_{fb}-2}\right| = \frac{1}{K_{fb}-2} \Rightarrow \frac{1}{K_{fb}-2} < 0.1 \Rightarrow \boxed{K_{fb} > 12}$$

(4) $\boxed{|K_{fb}| < 20}$

There is no K_{fb} that can satisfy the requirements (1), (2), (3), (4).

(f) Remove the assumption that $K_{ff} = 0$ and solve the control design in the previous question 3(e). In other words, design a feed-forward controller K_{ff} and feedback controller K_{fb} which meet the specifications (1)-(4) of the previous question 3(e). Report the values of K_{ff} and K_{fb}.

(1) $K_{fb} > 2$

(2) $y_{ss} = \frac{K_{fb} + K_{ff}}{K_{fb} - 2} r$

$$|y_{ss} - r| = \left| \frac{K_{ff} + 2}{K_{fb} - 2} r \right| = \frac{1}{K_{fb} - 2} \cdot |K_{ff} + 2||r|$$

Thus, $|y_{ss} - r| < 0.1|r|$ gives us the condition

$$\frac{1}{K_{fb} - 2} \cdot |K_{ff} + 2| \cdot \cancel{|r|} < 0.1 \cancel{|r|}$$

$$\Rightarrow \quad |K_{ff} + 2| < 0.1(K_{fb} - 2)$$

(3) $K_{fb} > 12$

(4) $|K_{fb}| < 20$

Any choice that satisfies the conditions (1), (2), (3), (4) is a valid choice.

I choose $K_{fb} = 14$ and $K_{ff} = -2$

4. Consider the same system of the previous exercise

$$\dot{y}(t) - 2y(t) = u(t)$$

and a controller described by the differential equation

$$\dot{u}(t) = K_1(r - y) + K_2\dot{y}$$

What are the conditions on K_1 and K_2 which result into a stable closed-loop system? (Hint: the closed-loop system linking r and y will be a second order differential equation)

$$\dot{y}(t) - 2y(t) = u(t)$$

$$\overset{d/dt}{\Rightarrow} \quad \ddot{y}(t) - 2\dot{y}(t) = \dot{u}(t)$$

$$= K_1(r-y) + K_2\dot{y}$$

$$\Rightarrow \quad \ddot{y}(t) - (K_2+2)\dot{y}(t) + K_1 y(t) = K_1 r(t)$$

characteristic polynomial,

$$s^2 - (K_2+2)s + K_1$$

is stable if $\boxed{\begin{array}{lll} -(K_2+2) > 0 & \Rightarrow & K_2 < -2 \\ K_1 > 0 & & \end{array}}$

due to RH2.

Alternatively, the roots are,

$$\lambda_{1,2} = \frac{K_2+2 \pm \sqrt{(K+2)^2 - 4K_1}}{2}$$

we need Real$(\lambda_{1,2}) < 0$ to be stable,

If $(K+2)^2 - 4K_1 < 0$, then $\lambda_{1,2}$ are complex

$$\text{Re}(\lambda_{1,2}) = \frac{K_2+2}{2} < 0 \quad \Rightarrow \quad K_2 < -2$$

$$K_1 > 0$$

If $(K+2)^2 - 4K_1 > 0$, then need $K+2 \pm \sqrt{(K+2)^2 - 4K_1} < 0$

$$\Rightarrow \quad K_2 < -2$$

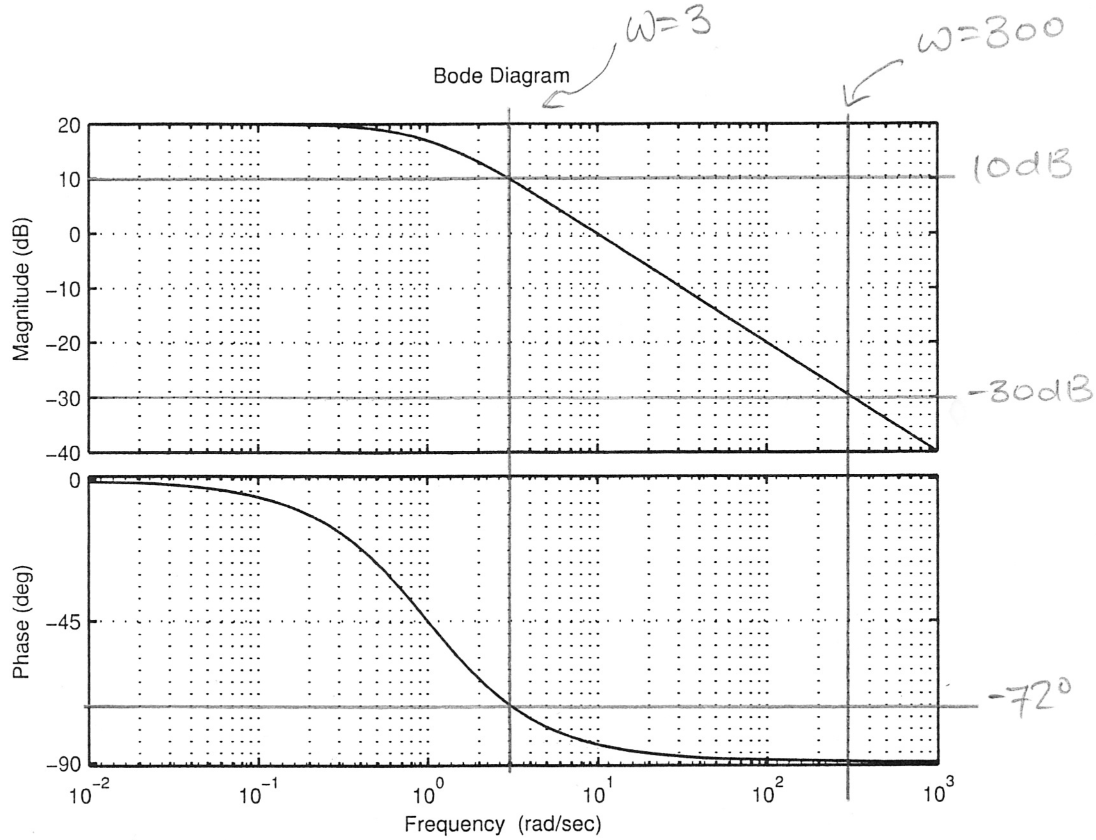
$$K_1 > 0.$$

5. Consider the first order LTI system described by the differential equation

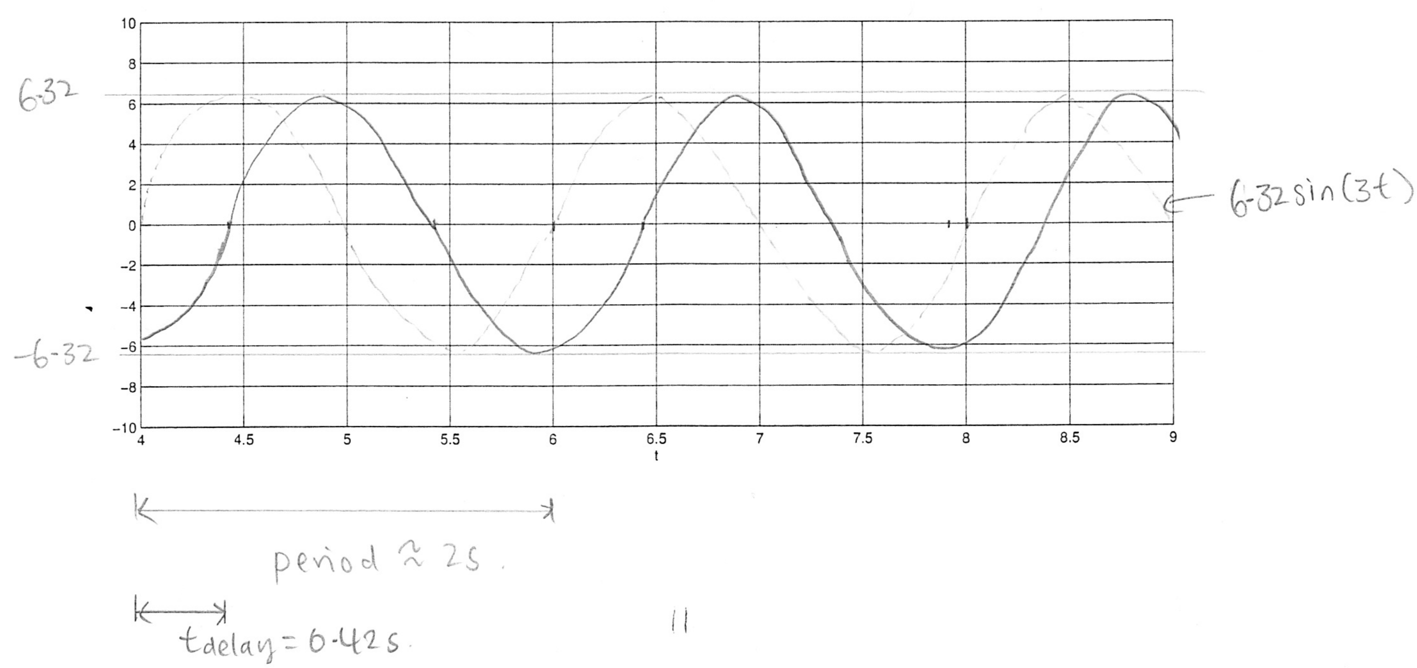
$$\dot{y}(t) + ay(t) = bu(t)$$

where $u(t) = \begin{cases} 0 & t < 0 \\ 2\sin(\omega t) + \sin(100\omega t) & t \geq 0 \end{cases}$ is the input, and $y(t)$ is the output.

The bode plot for this system is shown below.



(a) Assume that the output has reached steady state after 4 seconds, based on the bode plot, draw the steady-state output $y_{ss}(t)$ for $\omega = 3$. Make sure you label the response properly, i.e. amplitude, phase-shift, period.



① Read Bode plot for magnitude of gain and phase shift.

@ $\omega = 3$ and $\omega = 300$

② Convert Bode units.

$M(3)$: 10 dB $\Rightarrow$ $20 \log_{10} M(3) = 10$

$M(3) = 10^{1/2}$

$= 3.16$

$M(300)$: -30 dB $\Rightarrow$ $20 \log_{10} M(300) = -30$

$M(300) = 10^{-3/2}$

$= 0.03.$

③ we can ignore the effect of the $\sin(100\omega t)$ term in the input since this attenuated by 0.03. $\Rightarrow$ negligible.

④ compute the phase shift, time delay, period, amplitude.

$\phi(3) = -72^{\circ} = -1.26$ rad

$t_{delay} = \frac{-\phi(3)}{\omega} = \frac{1.26}{3} = 0.42$ s.

$\text{period} = \frac{2\pi}{\omega} = \frac{2\pi}{3} = 2.1$ s

≈ 2 s.

$\text{amplitude} = 2M(3) = 6.32$

$y(t) = 6.32 \sin(3t - 1.25)$

⑤ since period $\approx$ 2s, start @ 4s