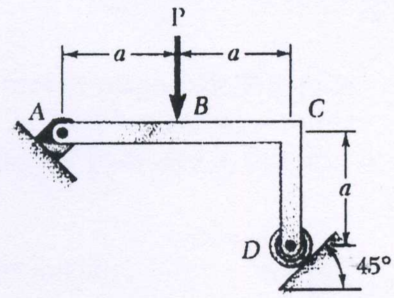
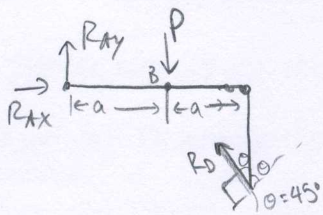
Name Caleb Boyd SID [redacted]

+20

1. (20 points) The massless, rigid structure ABCD is loaded at point B by a vertical force of magnitude P. Determine the reaction forces at pin A and roller D.



FBD



$\Sigma M_B = 0$

R_D has line of action through B, so does P & R_{Ax}

$R_{Ay} \cdot a = 0$

$R_{Ay} = 0$ ✓

$\Sigma F_y = 0$ $\qquad$ $\Sigma F_x = 0$

$R_{Ay} + R_D \cos\theta - P = 0$ $\qquad$ $R_{Ax} - R_D \sin 45° = 0$

$R_D \cos\theta = P$ $\qquad$ $R_{Ax} = P\sqrt{2} \cdot \frac{1}{\sqrt{2}} = P$

$\cos 45° = \sin 45° = \frac{1}{\sqrt{2}}$ $\qquad$ $R_D = \frac{P}{\cos 45°}$

$R_D = P\sqrt{2}$ ✓

$$\boxed{\begin{array}{l} R_{Ay} = 0 \\ R_{Ax} = P \rightarrow \\ R_{Dy} = P \uparrow \\ R_{Dx} = P \leftarrow \end{array}}$$

✓ ✓

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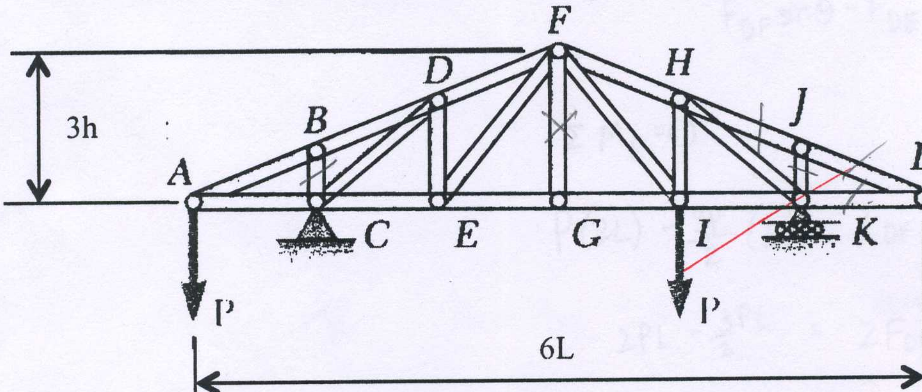
30

2. (30 points total) The truss shown below is loaded by vertical forces of magnitude P at joints A and I. It is supported by a pin at joint C and a roller at joint K. All of the horizontal members of the truss are of length *L*, and the vertical member BC has length *h*. For purposes of this analysis, all members may be treated as massless.

(a) (5 points) Identify any zero-force members that exist for this loading.

(b) (10 points) Determine the reaction forces at C and K.

(c) (15 points) Determine the forces in members CE and DE. Be sure to indicate clearly whether each member is in tension or compression.



a) at L: FBD F_{JL}, F_{KL}

$\Sigma F_y = 0$ $\boxed{F_{JL} = 0}$

$\Sigma F_x = 0$ $\boxed{F_{KL} = 0}$

at B: FBD F_{BD}, F_{AB}, F_{BC}

$\boxed{F_{BC} = 0}$

at J: FBD F_{HJ}, F_{JK}

$\Sigma F_x = 0$ $\boxed{F_{HJ} = 0}$

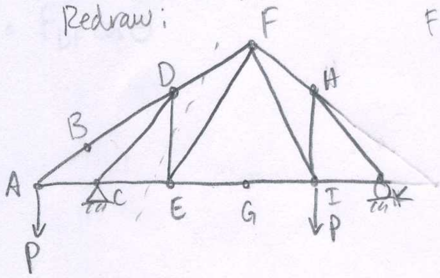
$\boxed{F_{JK} = 0}$

at H: FBD F_{FH}, F_{HK}, F_{HI}

at G: FBD F_{FG}, F_{EG}, F_{GI}

$\boxed{F_{FG} = 0}$

Redraw: FBD



FH ~~X~~ H

b) $\Sigma F_x = 0$

$\boxed{R_{Cx} = 0}$

$\Sigma F_y = 0$

$R_{Cy} + R_{Ky} - P - P = 0$

$R_{Cy} + R_{Ky} = 2P$

$\Sigma M_C = 0$

$P(L) + R_{Ky}(4L) - P(3L) = 0$

$4 R_{Ky} \cancel{L} = 2P\cancel{L}$

$\boxed{R_{Ky} = \frac{P}{2} \uparrow}$

$\boxed{R_{Cy} = \frac{3P}{2} \uparrow}$

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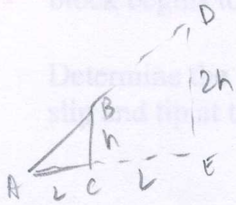
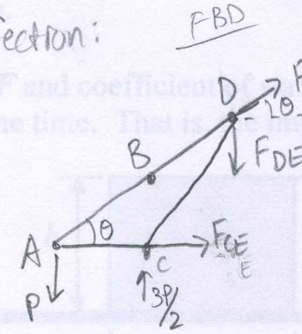
Problem 2 (continued)



Section: FBD



$\tan\theta = \frac{h}{L}$

$\Sigma F_x = 0$

$F_{DF}\cos\theta + F_{CE} = 0$

$\Sigma F_y = 0$

$F_{DF}\sin\theta - F_{DE} - P + \frac{3P}{2} = 0$

$F_{DF}\sin\theta - F_{DE} = -\frac{P}{2}$

$\Sigma M_E = 0$

$P(2L) - \frac{3P}{2}(L) - F_{DF}\cos\theta(2h) = 0$

$2PL - \frac{3}{2}PL = 2F_{DF}\cos\theta\, h$

$\frac{1}{2}PL = 2F_{DF}\cos\theta\, h$

$\frac{PL}{4h} = F_{DF}\cos\theta$

$\frac{PL}{4h\cos\theta} = F_{DF}$

$\frac{PL}{4h} + F_{CE} = 0$

$F_{CE} = -\frac{PL}{4h}$

$\boxed{F_{CE} = \frac{PL}{4h} \text{ compression}}$

$F_{DF}\sin\theta - F_{DE} = -P/2$

$\frac{PL}{4h}\tan\theta - F_{DE} = -P/2$

$-F_{DE} = -\frac{P}{2} - \frac{PL}{4h}\cdot\frac{h}{L}$

$\boxed{F_{DE} = \frac{P}{2} + P/4 = \frac{3P}{4} \text{ tension}}$

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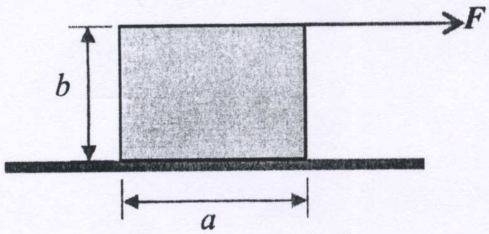
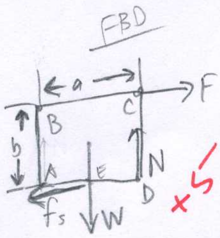
3. (25 points) Consider a block of mass m resting on a rough horizontal surface, with coefficient of static friction μ_s between the block and the surface. A horizontal force $\boldsymbol{F}$ is applied at the upper right-hand corner of the block. The magnitude of this force is slowly increased until the block begins to move.

25/25

Determine the force $\boldsymbol{F}$ and coefficient of static friction μ_s that will cause in the block to begin to slip and tip at the same time. That is, the impending motion is simultaneous slipping and tipping.





$W = mg$ $\quad g = 9.81\ m/s^2$

For sliding:

$F \geq f_s$ Point when $F = f_s$ is impending motion.

$f_s = \mu_s N$

For tipping:

N will be at far left corner of the block, creating its largest possible moment. Tipping is impending when $\Sigma M = 0$ and N is at this point.

$\Sigma F_x = 0$

$F - f_s = 0$

$F - \mu_s N = 0$

$F - \mu_s mg = 0$

$F = \mu_s mg$ +4

$\Sigma F_y = 0$

$N - W = 0$

$N = W$

$N = mg$ +6

$\Sigma M_C = 0$ +6

$W(a/2) - f_s(b) = 0$

$W(a/2) = f_s b$

$\frac{mga}{2} = \mu_s N b$

$\frac{\cancel{mg}a}{2} = \mu_s \cancel{mg} b$

$\boxed{\frac{a}{2b} = \mu_s}$ +1

$\boxed{F = \frac{mga}{2b}}$ +1

Check:

$\Sigma M_E = 0$

$N(a/2) - F(b) = 0$

$\frac{mga}{2} - \frac{mga}{2b} \cdot b = 0$

$\checkmark \ \frac{mga}{2} - \frac{mga}{2}$

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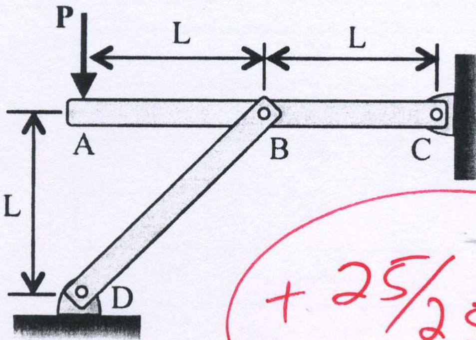
4. (25 points) The massless frame shown consists of two rigid members, ABC and BD, and is loaded at point A by a vertical force **P**.

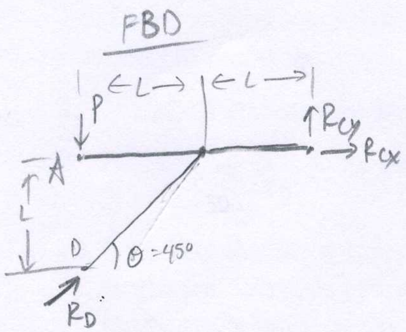
Determine the reaction force at point C, written in terms of its horizontal and vertical components. Be sure to clearly identify the direction (right/left, up/down) that component acts.



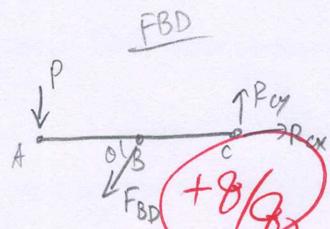
+25/25

BD is a two force body, forces will be ∥ along its length

FBD



FBD



+8/8

$\Sigma F_x = 0$

$R_{cx} - F_{BD}\cos\theta = 0$

$R_{cx} = F_{BD}\cos\theta$

$\Sigma F_y = 0$

$R_{cy} - P - F_{BD}\sin\theta = 0$

$R_{cy} = P + F_{BD}\sin\theta$

$\Sigma M_c = 0$

$P(2L) + F_{BD}\sin\theta\,(L) = 0$

Because $\theta = 45°$, $F_{BD}\sin\theta = F_{BD}\cos\theta$

+15/15

$F_{BD}\sin\theta\,(\cancel{L}) = -P2\cancel{L}$

$F_{BD}\sin\theta = -2P$

$R_{cx} = F_{BD}\cos\theta$

$R_{cx} = -2P$

$R_{cy} = P + F_{BD}\sin\theta = P - 2P = -P$

+2/2

$$\boxed{\begin{array}{l} R_{cx} = 2P \leftarrow \\ R_{cy} = P \downarrow \end{array}}$$