CE 111: Environmental Engineering
Midterm Exam #2 (31 October 2012)

Name: SOLUTION

GROUND RULES: This is a closed-book/closed-note exam, except that you are permitted two sheets of notes. Do your work on the paper provided. Please be sure that your name is written on each page you submit. The total score possible is 20 points, and the time allowed is 50 minutes. Use the time wisely. Good luck!

REMINDER: Read the questions carefully and respond appropriately. Partial credit is granted in most places if your work can be understood (and your thinking is reasonable). All of the information below is true and some of it may be useful.

#1 (5 max.) ______ #2 (5 max.) ______ #3 (10 max.) ______

Total Score (20 max.): ☐

DATA AND RELATIONSHIPS:

Composition, dry atmos.:	N_2 = 78%, O_2 = 21%, Ar = 1%
Ideal gas law:	$PV = nRT$; $P_iV = n_iRT$
Gas constant:	$R = 0.082$ atm mol^{-1} L K^{-1}
Mole fraction:	$y_i = n_i/n = P_i/P$
Pressure in troposphere:	$P(z) = P_o \exp(-z / 7.3\text{km})$

Reaction kinetics for A → products:

0th order	$d[A]/dt = -k$
1st order	$d[A]/dt = -k[A]$

$dC/dt = S - LC$ with constant S and L, and $C(t = 0) = C_o$

Solution: $C(t) = C_o \exp(-L\,t) + (S/L)[1 - \exp(-L\,t)]$

Conversion factors:

volume:	1 m^3 = 1000 L
volume:	1 L = 1000 cm^3
temperature:	T (K) = T (°C) + 273
pressure:	1 atm = 101.3 kPa

Flux (amt per area per time):

Interfacial mass tr.	$J_{aw} = k_{aw}(C_s - C)$
Diffusive	$J_d = -D\,dC/dx$
Advective	$J_a = u\,C$
Settling	$J_g = v_t\,C$

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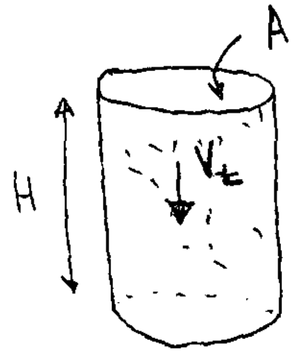
1. Gravitational Settling: Mixed versus Unmixed (5 points)

Consider two batch reactors. Each reactor is a right-circular cylinder of volume V = AH, where A is the horizontal cross-sectional area and H is the height of the fluid in each reactor.

The reactors are initially filled with a uniform suspension of particles of concentration N_o/V, where N_o is the initial number of suspended particles. The particles settle under the influence of gravity. All of the particles have the same settling velocity, v_t. The reactors are identical *except that one is completely mixed and the other is unstirred.*

The fluid in the unstirred reactor is motionless. Assume that the mixing in the mixed reactor is sufficient to maintain a uniform particle concentration in the fluid, but not so vigorous as to resuspend particles once they settle to the bottom (i.e., particles that settle to the bottom are irreversibly removed from the fluid).

When the concentration of particles decreases to zero in one reactor, what is the concentration of particles in the other reactor?



$$J_s = V_t C = V_t \frac{N}{V}$$

$$\frac{dN}{dt} = -J_s A = -V_t \frac{NA}{V}$$

For unmixed reactor, all particles settle in time $= \frac{H}{V_t}$

For mixed reactor: $N(t) = N_0 \exp\left(-V_t \frac{A}{V} t\right)$

evaluate @ $t = \frac{H}{V_t}$: $N\left(\frac{H}{V_t}\right) = N_0 \exp\left(-V_t \frac{A}{V} \frac{H}{V_t}\right)$

Since $AH = V$, $N\left(\frac{H}{V_t}\right) = N_0 \exp(-1) = 0.37 N_0$

$\therefore$ @ $t = \frac{H}{V_t}$, conc'n. particles in $\boxed{\text{mixed reactor}}$ equals $\boxed{0.37 \frac{N_0}{V}}$

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Explicit solution for unmixed reactor

$$\frac{dN}{dt} = -V_t \frac{N_0}{V} A$$

← rate is const. b/c conc^n. particles just above reactor bottom is const. till all particles have settled.

$$\int_{N_0}^{N(t)} dN = -V_t \frac{N_0}{V} A \int_{0}^{t} dt$$

$$N(t) - N_0 = -V_t \frac{N_0}{V} A t - 0$$

$$N(t) = N_0 - V_t \frac{N_0}{V} A t$$; evaluate @ $t = \frac{H}{V_t}$

$$N\left(\frac{H}{V_t}\right) = N_0 - V_t \frac{N_0}{V} \frac{A H}{V_t} = N_0 - N_0 = 0$$

So @ $t = \frac{H}{V_t}$ all particles have settled in unmixed reactor, so conc^n. particles $= 0$.

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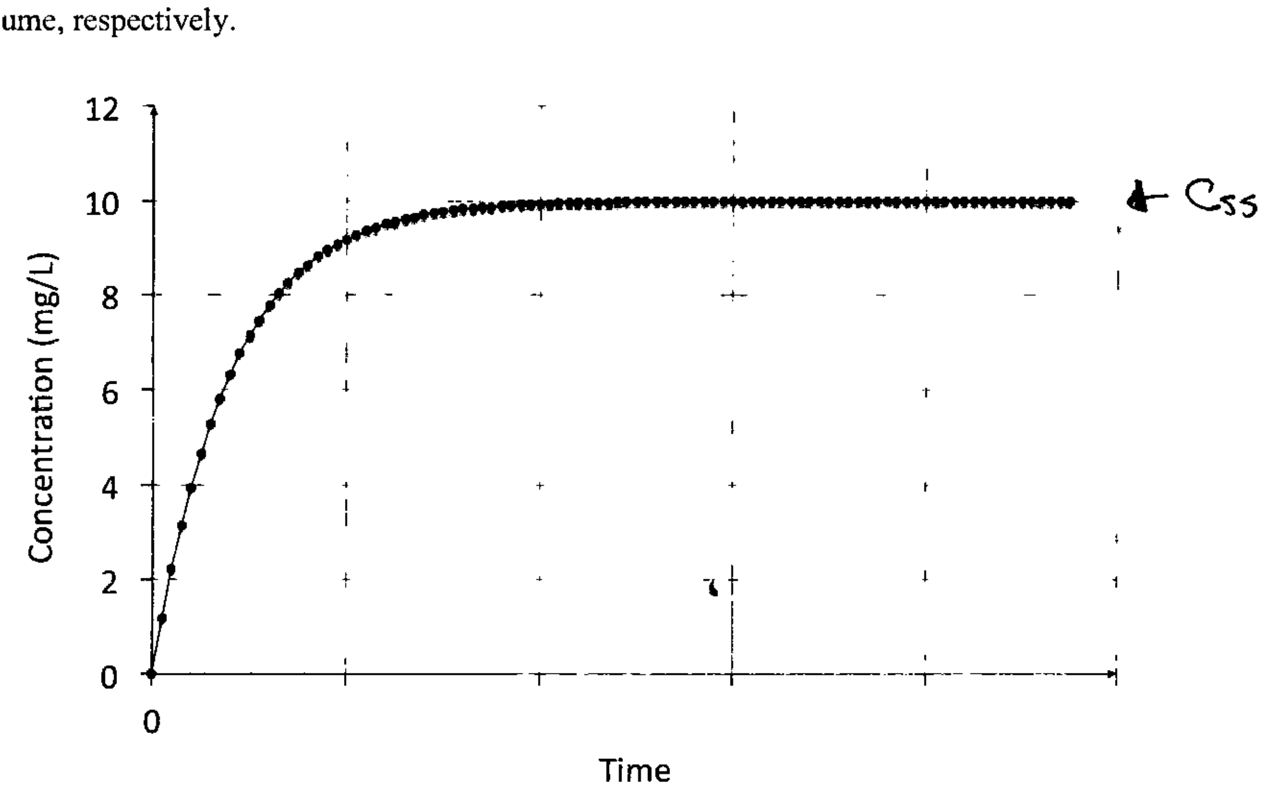
2. Continuously Mixed Flow Reactor (5 points, 2.5 points each)
The concentration of a contaminant in CMFR is plotted below for $t \geq 0$. The contaminant concentration at the inlet of the reactor is $C_{in} = 0$ for $t < 0$. At $t = 0$, C_{in} suddenly jumps to a value that is greater than zero and remains at that level indefinitely. The contaminant is consumed in the reactor by a chemical reaction that is first order in the contaminant concentration, i.e., $r = -kC$. The value of the rate constant $k = 0.25Q/V$, where Q and V are the reactor volumetric flow rate and volume, respectively.



(a) Determine the inlet concentration, C_{in}, in mg/L units for $t \geq 0$.

CMFR eqn: $V\frac{dC}{dt} = QC_{in} - QC - kCV$

Graph indicates $C_{ss} = 10\,\text{mg/L}$

@ S.S., $QC_{in} = (Q + kV)C_{ss}$

$C_{in} = \left(1 + \frac{kV}{Q}\right)C_{ss}$; $k = 0.25\,Q/V$

$C_{in} = \left(1 + 0.25\frac{Q}{V}\frac{V}{Q}\right)C_{ss} = (1 + 0.25)\,C_{ss}$

$$\boxed{C_{in} = 12.5\,\text{mg/L}}$$

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(b) What fraction of the mass of the contaminant entering the CMFR is removed by the chemical reaction rather than the flow out of the CMFR?

mass removal rate by rxn $= kCV$

" " " " flow $= QC$

$\therefore$ fraction removed by rxn $= \dfrac{kCV}{kCV + QC}$

$$= \frac{1}{1 + \frac{Q}{Vk}} \quad ; \quad Q = 4kV$$

$$= \frac{1}{1 + \frac{4kV}{Vk}}$$

$$= \underline{\frac{1}{5} \text{ or } 0.20 \text{ or } 20\%}$$

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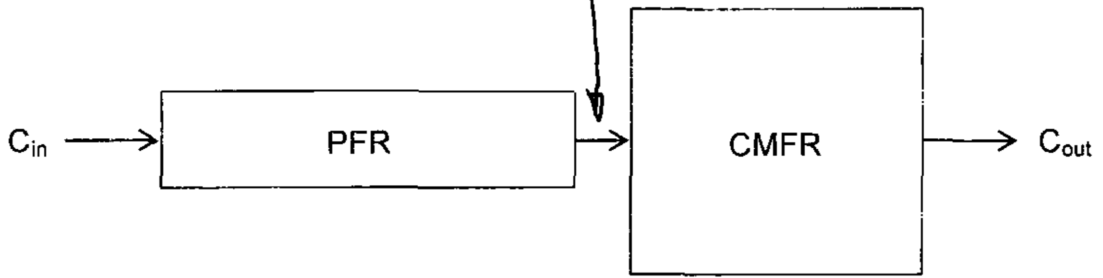
3. Interfacial Mass Transfer (10 points)

Part 1: Reactors in Series (5 points)
Water flows through a PFR and immediately into a CMFR, as illustrated below. The water contains a volatile contaminant. Both reactors are open to the atmosphere, and the contaminant is removed from each reactor due to volatilization. Assume that the contaminant flux across the air-water interface follows the two-film resistance model, where $J_{aw} = k_{aw}(C_s\text{-}C)$, and that the contaminant is not present in the atmosphere. The value of the interfacial mass transfer coefficient $k_{aw} = 0.5Q/A_s$ for both reactors, where Q/A_s is the ratio of the volumetric flow rate to the surface area of each reactor.

Determine the steady state outlet concentration, C_{out}, in mg/L units if the inlet concentration, C_{in}, is 100 mg/L.

$C_{out,PFR} = C^* = C_{in,CMFR}$



Think of batch reactor traveling along PFR.

$$\frac{dc}{dt} = J\frac{A_s}{V} = k_{aw}\frac{A_s}{V}(C_s - C) \quad ; \quad C_s = \emptyset$$

$$\frac{dc}{dt} = -k_{aw}\frac{A_s}{V}C$$

$$\int_{C_{in}}^{C^*}\frac{dc}{C} = -\frac{k_{aw}A_s}{V}\int_0^{\theta} dt$$

θ ← exits PFR @ $t = \theta$ hydraulic res. time

$$C^* = C_{in}\exp\left(-k_{aw}\frac{A_s}{V}\theta\right) \quad ; \quad \theta = \frac{V}{Q}$$

$$C^* = C_{in}\exp\left(-k_{aw}\frac{A_s}{Q}\right) \quad ; \quad k_{aw} = 0.5\,Q/A_s$$

$$C^* = C_{in}\exp(-0.5) = \left(100\frac{mg}{L}\right) = 60.6\ mg/L$$

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CMFR equation: $V\frac{dC}{dt} = QC^* - QC + JA_s$

@ S.S. and inserting $J = -k_{aw}C$

$$QC^* = QC + k_{aw}A_sC = (Q + k_{aw}A_s)C$$

$$C = \frac{QC^*}{Q + k_{aw}A_s}$$

$$C = \frac{C^*}{1 + k_{aw}\frac{A_s}{Q}} \quad ; \quad k_{aw} = 0.5Q/A_s$$

$$C = \frac{C^*}{1.5} = \frac{60.6\,mg/L}{1.5}$$

$$\boxed{C = 40.4\ mg/L}$$

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Part 2: Concepts in Flux (5 points, 1 point each)
The flux of species across an air-water interface can be described by the following equation:

$J_{aw} = k_{aw}(C_s - C)$

(a) Provide a clear definition of the term C_s. = aqueous concn. in bulk when bulk is in equilibrium with bulk air concentration.

(b) Is the following statement true or false? For a highly soluble species, the overall mass-transfer coefficient is equal to the mass-transfer coefficient through the water film layer near the air-water interface.

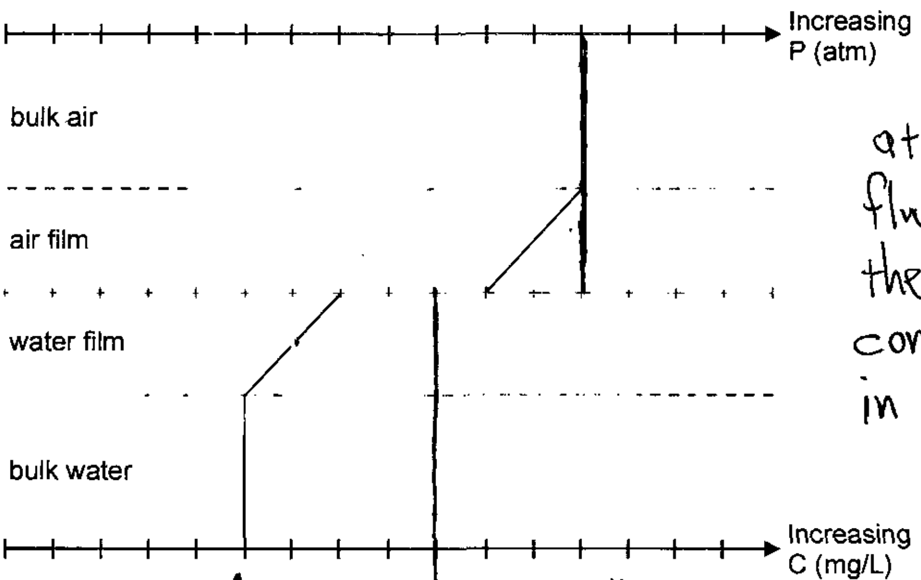
False

(c) The figure below shows the idealized concentration profiles for a species near the air-water interface, according to the two-film resistance model. Does the figure correspond to net volatilization or dissolution? Dissolution. Diffusion results in net transfer of material from regions of hi to low concn.

(d) Does equilibrium prevail anywhere in the figure? If yes, where?

Yes, at air-water interface

(e) Assume the partial pressure in the bulk air is constant. Add to the figure the concentration profile when the rates of material transfer from the air to the water and from the water to the air are equal.



at equilibrium flux = 0, therefore no concn. gradient in either film

J>0, net flux from air to H_2O ~> $C < C_s$

C_s ← equilib. concn. > initial concn.