Physics 7C Section 1
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Name:________________________________ SID:____________________

Choose four out of the five problems proposed, the test duration is 180 minutes.

1. Consider Cherenkov radiation emission by a charged particle. i) Using the Huygens principle determine the relation between the Cherenkov angle θ_c and the velocity β of the particle, where θ_c is defined as the angle between the direction of propagation of the Cherenkov photons and the particle direction. ii) A kaon particle K^- of mass $m_K = 498$ MeV/c^2 travels through a thick slab of glass, of refractive index $n = 1.5$. Determine the minimum energy for the kaon to emit Cerenkov radiation. iii) Assume that a kaon particle with energy corresponding to the Cherenkov emission threshold, determined above, decays according to the process $K^- \rightarrow \pi^- \pi^0$ inside the glass, determine whether the π^- particle will also emit Cherenkov light and, if so, determine the value of θ_c for the π^-. ($m_{\pi^-} = 138$ MeV/c^2 and $m_{\pi^0} = 140$ MeV/c^2).

2. Consider the scattering of a photon, γ, of energy E_γ on an electron e^- of mass $m_e = 0.511$ MeV/c^2 initially at rest, $\gamma e^- \rightarrow \gamma e^-$. i) Using energy and momentum conservation determine the energy transferred by the photon to the electron in the reaction as a function of the scattering angle θ of the photon from its original trajectory. ii) Assuming that the photon energy before the scattering, E_γ, is 1 MeV, determine for which photon scattering angles θ the electron will gain enough energy to become relativistic, i.e. for its final velocity β to exceed 0.3. iii) Compute the invariant mass of the final state γe^- system, after the scattering, as a function of θ and compare it to the invariant mass of the system before scattering.

3. Consider a set of N electrons ($m_e = 0.511$ MeV/c^2) confined in a 1-D square well defined by $U(x) = {}^{0 \;\; \text{for } 0<x<L}_{U_0 \;\; \text{elsewhere}}$, i) solve the Schrodinger equation for this system, determine the eigenfunctions including the normalization coefficient and its energy levels in the limit $U_0 \rightarrow \infty$. ii) Taking into account that electrons have spin $\frac{1}{2}$ and the Fermi exclusion principle, use the result of point i) to estimate the number N_F of electrons that can be placed in the well on energy levels below U_0=70 eV, if L=0.3 nm. iii) Discuss how N_F changes if the square well is in 3-D with $L_x = L_y = 2L_z = L$. (Use $hc = 1240$ eV nm)

4. Consider an electron of mass m_e under the influence of a Coulomb potential $U(r) = -\frac{Ze^2}{r}$.
i) Write the Schrodinger equation for the electron in parabolic coordinates: $\xi = r - z$, $\eta = r + z$, $\phi = atan\frac{y}{x}$, where $r = \sqrt{x^2 + y^2 + z^2}$, and demonstrate that the solution $\Psi(\xi, \eta, \phi)$ can be factorised as $\Psi(\xi, \eta, \phi) = \Psi_1(\xi)\Psi_2(\eta)\Psi_3(\phi)$, similarly to the wavefunction $\Psi(r, \theta, \phi) = R(r)\Theta(\theta)\Phi(\phi)$ in polar coordinates. ii) Write the three Schrodinger equations in the ξ, η and ϕ coordinates.
(The Laplace operator $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ in parabolic coordinates is $\frac{4}{\xi+\eta}\frac{\partial}{\partial \xi}(\xi\frac{\partial}{\partial \xi}) + \frac{4}{\xi+\eta}\frac{\partial}{\partial \eta}(\eta\frac{\partial}{\partial \eta}) + \frac{1}{\xi\eta}\frac{\partial^2}{\partial \phi^2}$)

5. The hydrogen atom is kept together by the electromagnetic force. Consider an "hydrogen" atom where the only attractive potential is the gravitational potential between the electron and the proton $V(r) = \frac{G m_e m_p}{r}$, with $G = 6.7 \times 10^{-39}\ \hbar c\ (\text{GeV}/c^2)^{-2}$, $m_e = 0.511$ MeV/c^2, $m_p = 938$ MeV/c^2. i) Write the radial part of the Schrodinger equation for the state with the lowest quantum numbers, ii) determine its energy and the most probable value of the orbit radius, iii) compare these results to those of the H atom. (Remember that $\hbar c = 197.3$ MeV fm, 1 fm = 10^{-15} m, for the H atom $\Psi_{100} = R_{1,0}Y_{0,0}$ with $Y_{0,0} = \frac{1}{2\sqrt{\pi}}$, $R_{10} = \frac{2}{\sqrt{a_0^3}}e^{-r/a_0}$, $a_0 = \frac{\hbar}{k m_e e^2} = 0.0529$ nm)