

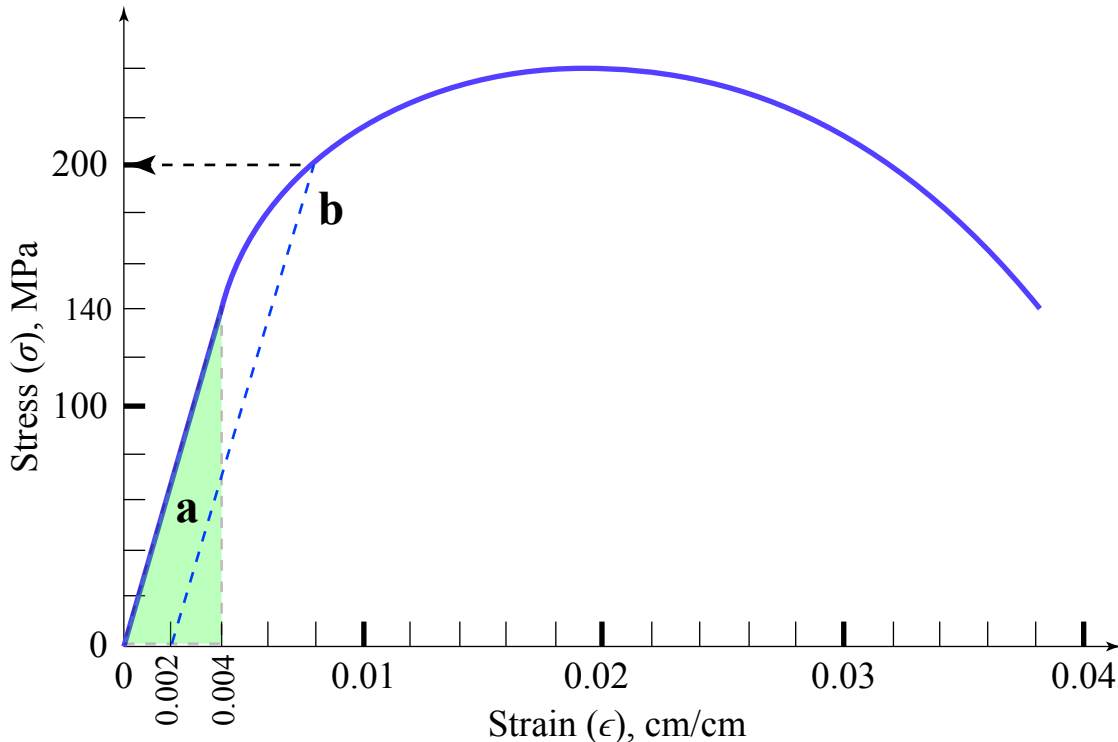
# Midterm 01 Solutions

## INSTRUCTIONS

- TIME.....Do not open pages until “START” is announced. This is a timed, 50 minute examination with 5 equally weighted problems.
- TRUST .....The engineering profession demands strict ethical standards of honesty and integrity. Engineers do not cheat on the job, and there will be no cheating on this exam.
- TOOLS.....Only *writing instruments / eraser / straightedge* are allowed. Remove all other materials, including books / reference materials / calculators / PDAs / cell phones (disable all sounds) / other electronic devices / headphones / ear buds / hats from your person / workplace.
- TEMPERAMENT .....Asking and answering questions during the exam are disruptive and discourteous to your classmates. So there will be *no questions* during the exam. Instead, please write your concerns or alternative interpretations in your answers.

## 1. Mechanical Behavior of Materials

The stress-strain curve shown below was obtained from a uniaxial tensile test performed on an aluminum alloy plate destined for application as an aircraft aileron trim component.



- a. The specified elastic modulus for this component is “70 to 85 GPa.” Does the alloy tested here fall within spec? Explain.

The elastic modulus ( $E$ ) is defined by Hooke’s Law ( $\sigma = E\epsilon$ ), which applies to the initial linear portion of the stress ( $\sigma$ ) vs. strain ( $\epsilon$ ) curve, where the **slope**  $E \equiv \sigma/\epsilon$ . Substituting values from the curve above and solving,

$$E = \frac{[140 - 0] \text{ MPa}}{[0.004 - 0] \text{ cm/cm}} = 35 \text{ GPa}$$

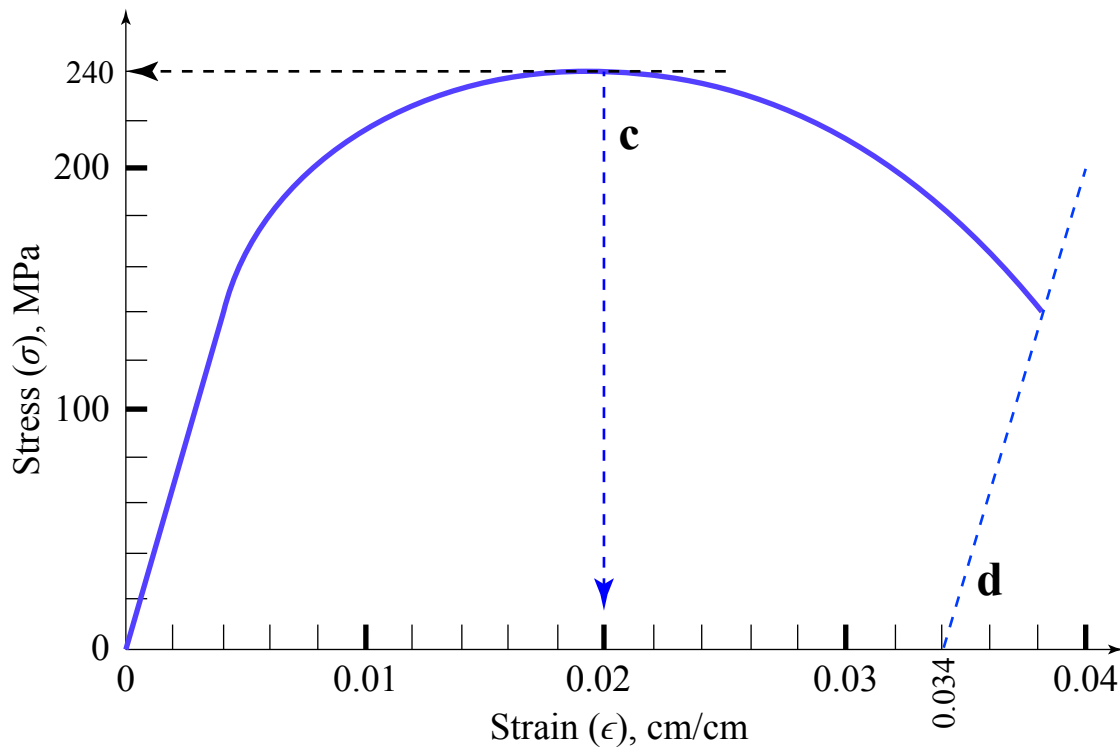
Consequently the answer is **NO**, the alloy **falls outside of spec**, achieving only half of its minimum required elastic modulus.

- b. At what value of stress did dislocations nucleate and glide in your test sample? Explain.

Dislocations are associated with plastic deformation, so they can only nucleate and glide at stress values **greater than the yield stress**. Using the 0.2% offset method as is the convention for aluminum alloys, a line with slope  $E$  originating at  $\epsilon = 0.002 \text{ cm/cm}$  intersects the curve at the defined yield strength. The answer is  $\sigma > 200 \text{ MPa}$ .

## 1. Mechanical Behavior of Materials

(same plot reproduced here)



- c. At what value of strain did “necking” initiate in your sample? Explain.

Necking occurs when the stress level reaches its ultimate value, or the peak in the engineering stress strain curve given here, shown to be 240 MPa.

The answer to this question is the corresponding **strain** at the onset of necking, which can be read directly from the plot,  $\epsilon = \mathbf{0.02 \text{ cm/cm}}$ .

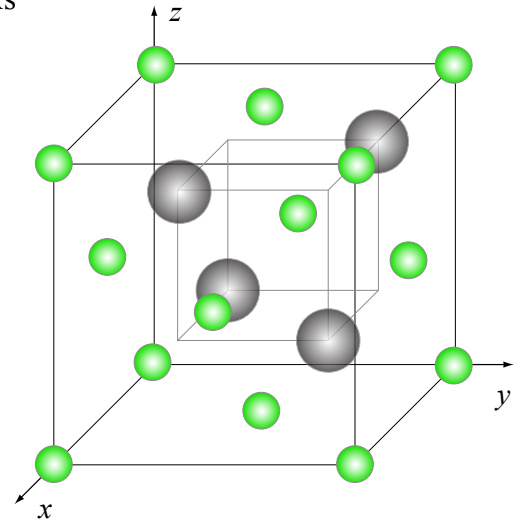
- d. Another critical requirement for the aileron trim component is ductility, specified to be “no less than 3.5% elongation at failure?” Did your test sample meet this spec? Explain.

The percent elongation at failure is the amount of **permanent strain** evident in a failed component, after elastic recovery has occurred. Elastic recovery is always shown by a line with slope  $E$  parallel to the initial linear portion of the curve. Constructing such a line originating at the point of failure and extending to the abscissa reveals that the permanent plastic strain is 0.034 cm/cm, or **3.4 %**. So again the answer is NO. The test sample **did not meet its ductility requirement**.

## 2. Bonding in Engineering Materials

- a. What makes covalent bonding in the semiconducting alloy GaAs “directional?” Explain.

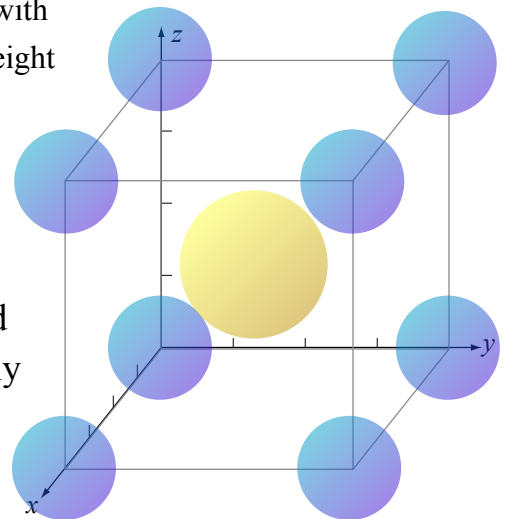
Covalent bonding occurs by a special type of electron sharing most often described as the **overlapping of valence electron orbitals**. Where overlap occurs, electron density is increased, enhancing bond strength and inducing a corresponding directionality in covalent bonding. The **directions along which orbital overlap is greatest are those that form the strongest bonds**. In semiconductors, the amount of orbital overlap is increased by  $sp^3$  hybridization, causing the **tetrahedral coordination** (CN = 4) found in GaAs.



- b. Ionically-bonded cesium chloride has  $\text{Cl}^-$  at the center of a cube with  $\text{Cs}^+$  at all eight corners. Alternatively, each  $\text{Cs}^+$  ion is bonded to eight  $\text{Cl}^-$  ions, also along the directions of the cube diagonals. Why is this not considered “directional” bonding?

Although this structure appears to be induced by directional bonding, we know that ionic bonding is **not** directional. What induces the structure exhibited by CsCl with its coordination number CN=8 is simply **space filling**, the packing of small cations and large anions to preserve charge neutrality at highest density. At specific **ratios of ionic radii** the coordination numbers of ionic solids **change**.

Directionally bonded solids like GaAs and pure Si have a fixed CN = 4, even as their atomic radius ratios vary. The structure of CsCl is therefore considered a **consequence of packing geometry, not bond directionality**.

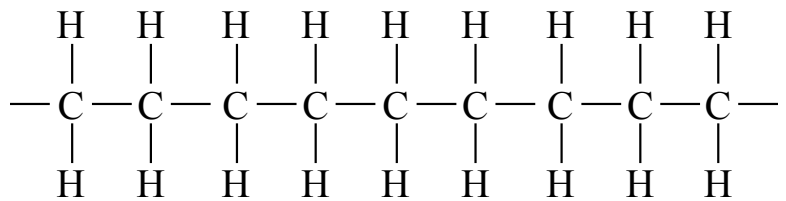


## 2. Bonding in Engineering Materials

- c. Metallically bonded copper is best known as an excellent electrical conductor. If a “sea of electrons” is required for “bonding” Cu atoms together, why don’t copper wires disintegrate when their electrons “wash away” through an electrical circuit under an applied voltage? Explain.

The motion of valence electrons under an applied electrical voltage is the reason for electrical conductivity, but **not all bonding electrons are removed** in such currents. The electron distribution in Cu wiring continues to secure the bond strengths of the Cu atoms because the “circuit” continuously resupplies electrons from a “source” as they flow through the conductors comprising the circuit towards a “drain.” In the absence of a closed circuit, high fields can indeed extract electrons from the Cu, breaking bonds, and resulting in structural damage.

- d. Primary bonds are known as chemical bonds that occur between individual atoms or ions, while secondary bonds are known as physical bonds that occur between groups of atoms or ions.

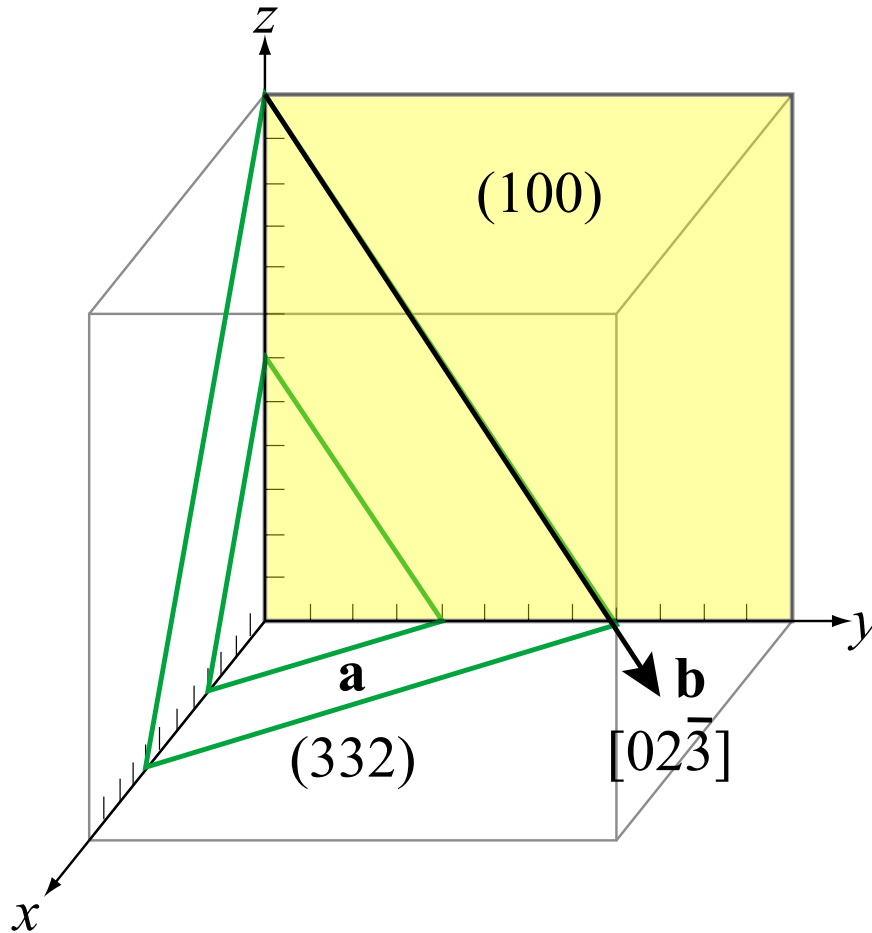


Explain how van der Waals bonds form and function in polyethylene.

Van der Waals bonds form in polyethylene (covalently bonded C and H in the chain configuration shown above) by the **dipolar attraction** between adjacent chains or coiled/folded segments of the same chain. The positively charged portions of the chain populated by hydrogen and negatively charged portions of an adjacent chain between the hydrogen atoms are attracted to one another. The function of such a bond is to give strength to the structure, enough to sustain a solid phase, but not strong enough to withstand even a small temperature rise, which causes severe softening of the structure, and a relatively low melting temperature.

### 3. Lattice Geometry

- a. Draw two members of the  $(332)$  family in the following unit cell.



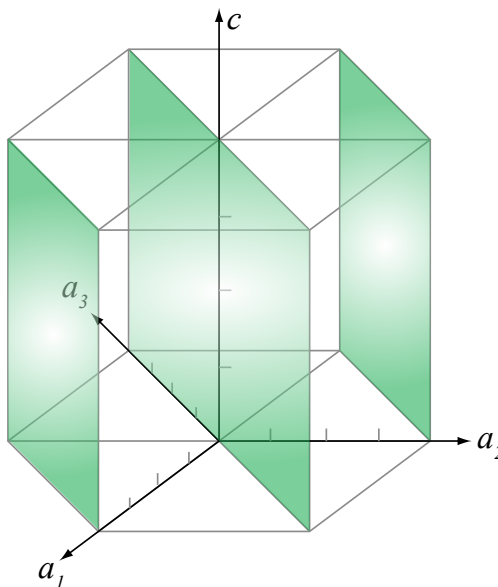
The  $(332)$  family is derived from fractional intercepts of  $\frac{1}{3}$  along the  $x$ -axis,  $\frac{1}{3}$  along the  $y$ -axis, and  $\frac{1}{2}$  along the  $z$ -axis, as **shown and labeled** on the drawing.

- b. Show and specify the lattice direction of the intersection between  $(332)$  and  $(100)$ .

The line of intersection between  $(332)$  and  $(100)$  is the  $[02\bar{3}]$  direction (or its negative), which is shown on the illustration to fall within the  $y$ - $z$  plane. It can be determined by noting the **coordinates of a point on the line** such as  $0, \frac{2}{3}, -1$ , and clearing fractions, or by taking the **vector cross product of the normals to the planes**  $[332] \times [100]$ , cross-checking with confirmation that the dot products between  $[02\bar{3}]$  and both plane normals  $[332]$  and  $[100]$  are zero.

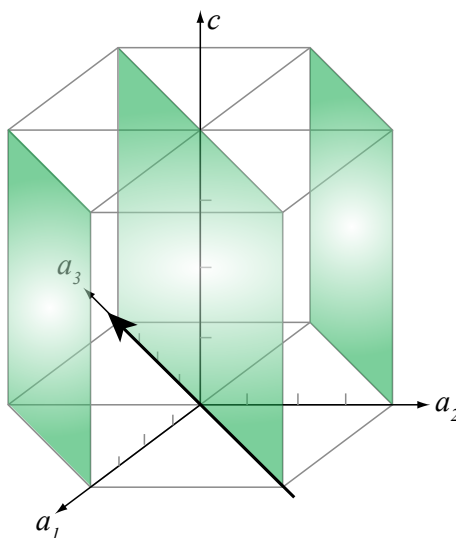
### 3. Lattice Geometry

- c. Draw three members of the  $(\bar{1}100)$  family in the following unit cell.



This family of planes follows the standard rules for Miller-Bravais notation, with intercepts  $-1$  along  $a_1$ ,  $+1$  along  $a_2$ ,  $\infty$  along  $a_3$  and  $\infty$  along  $c$ . All members of the family must be **parallel with identical interplanar spacings**, as drawn.

- d. Show and specify the lattice direction of the intersection between  $(\bar{1}100)$  and the basal plane.



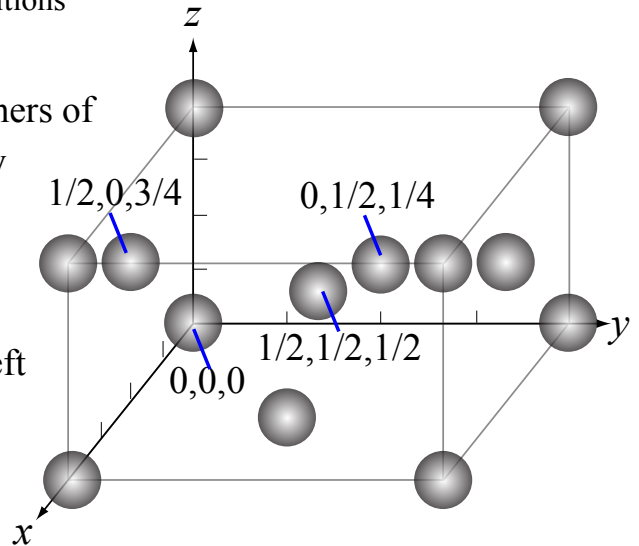
The line of intersection with the basal plane is fortuitously the  $a_3$  axis,  $[\bar{1}\bar{1}20]$ , which can also be expressed as its **negative**,  $[11\bar{2}0]$ . Either is correct.

## 4. Crystal Structure

Tin is a Group IV element that shares the diamond cubic structure of the Group IV members above it (C, Si, Ge), known as  $\alpha$ -tin, but only at relatively low temperatures,  $< 13^\circ\text{C}$ . At higher temperatures,  $\beta$  tin is the stable phase, and it has a tetragonal structure, with  $a = 5.83\text{\AA}$  and  $c = 3.18\text{\AA}$ . There are four Sn atoms per unit cell, at locations  $0,0,0$ ;  $1/2, 1/2, 1/2$ ;  $0, 1/2, 1/4$ ; and  $1/2, 0, 3/4$ .

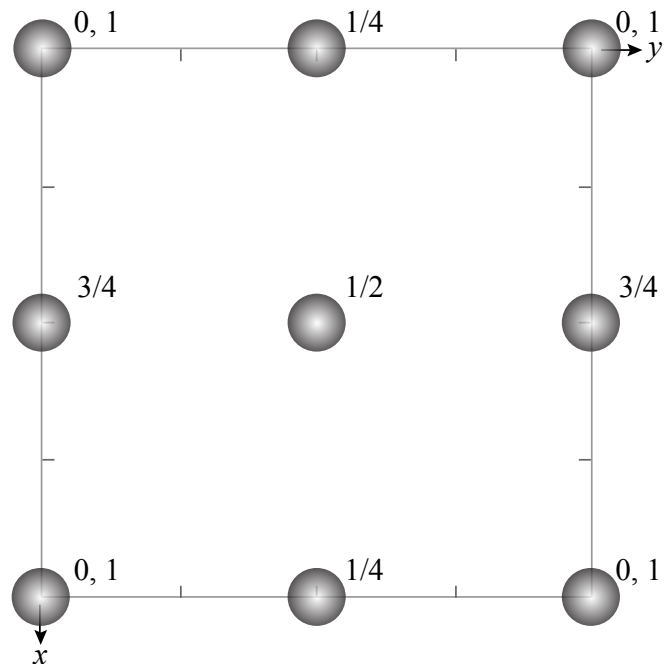
- a. Sketch the unit cell to the right indicating all atom positions within the cell.

The atom at  $0,0,0$  accounts for all eight corners of the unit cell, while the one at  $1/2, 1/2, 1/2$  is fully contained at its body center. The other atoms at  $0, 1/2, 1/4$  and  $1/2, 0, 3/4$  are both face-shared so they have pairs at opposing face locations (back face pairs with front face, left face with right face), as shown in the drawing.



- b. Sketch a  $c$ -axis projection of the same unit cell, and label the elevation of all Sn atoms from “0” at the bottom of the cell to “1” at the top of the cell.

The symmetry of the structure is more clearly revealed in this projection. Note that the atom at the body center position is **tetrahedrally coordinated** to the atoms on the face centers (two below [top and bottom in drawing] and two above [left and right] the center location), a remnant of the packing found in the lower temperature  $\alpha$  phase of Sn, and its Group IV relatives.





## 4. Crystal Structure

- c. Now use your sketches to identify an appropriate **Bravais lattice** and **motif** that defines this crystal structure?

Here are three possible options:

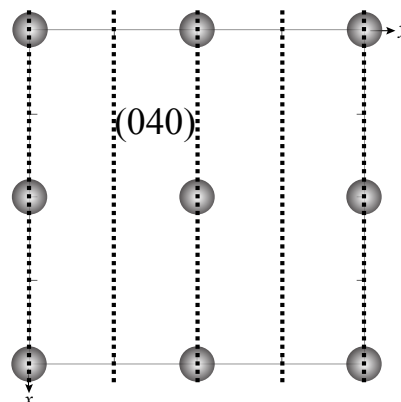
(1) a **simple tetragonal (ST)** Bravais lattice with a **4 atom** motif, at locations  $0,0,0$ ;  $\frac{1}{2},\frac{1}{2},\frac{1}{2}$ ;  $0,\frac{1}{2},\frac{1}{4}$ ; and  $\frac{1}{2},0,\frac{3}{4}$ . The ST lattice has a primitive unit cell containing a single lattice point, so the 4 atom motif accounts for the 4 atoms per unit cell.

(2) a **body-centered tetragonal (BCT)** Bravais lattice with a **2 atom** motif, at locations  $0,0,0$  and  $0,\frac{1}{2},\frac{1}{4}$  **or** (3) BCT with a 2-atom motif, at alternative locations  $0,0,0$  and  $\frac{1}{2},0,\frac{3}{4}$ . The BCT lattice has a non-primitive unit cell containing two lattice points, so the 2 atom motif accounts for the 4 atoms per unit cell. This is the **preferred choice** because it takes advantage of the symmetry of the BCT lattice for a more efficient specification of structure with a more compact motif.

- d. In a diffraction experiment, a peak is indexed as 040. Use your projection sketch above to relate this peak to the (040) family of planes in  $\beta$ -tin, commenting on how this peak is explained by Bragg's law,

$$n\lambda = 2d_{hkl} \sin\theta.$$

The projection sketch reveals that the (040) family of planes is **missing atoms**. There are  $S_n$  atoms on (020) planes, but every other (040) plane (dashed lines) is vacant. An 040 diffraction peak is therefore the result of **second order** ( $n = 2$ ) constructive interference among x-rays scattered by those atoms on (020) planes, consistent with Bragg's law.



This is also true for a 400 peak. Note however that there are atoms at  $\frac{1}{4}$  elevation (along  $z$ ) in the  $\beta$ -tin structure, and these populate the (004) family of planes. A diffraction peak of the type 004 would therefore be a first order ( $n = 1$ ) scattering condition from the (004) family of planes. This difference results from the tetragonal structure of Sn with  $a = b \neq c$ , all consistent with Bragg's law.

## 5. Crystal Defects

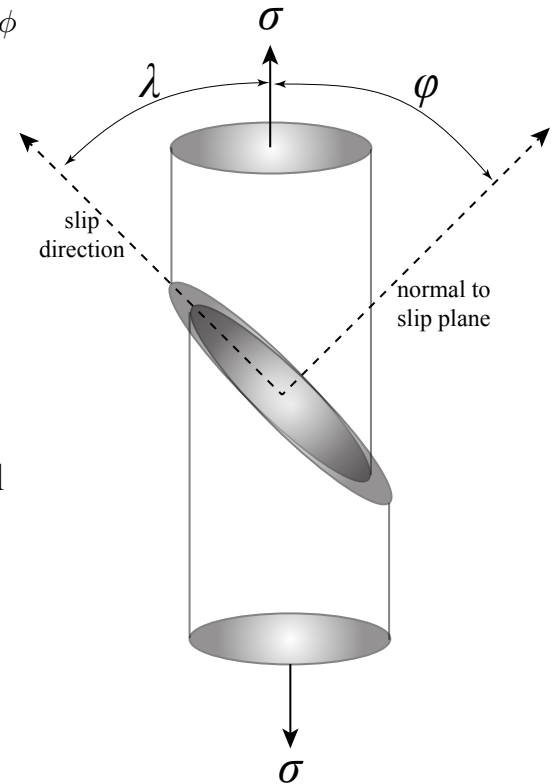
Dislocations will “slip” or “glide” when the applied stress ( $\sigma$ ) resolved into the slip plane ( $hkl$ ) in the slip direction  $[uvw]$  exceeds a critical value, given by

$$\tau_C = \sigma_C \cos \lambda \cos \phi$$

where  $\lambda$  and  $\phi$  are defined in the drawing.

- a. What is the required critical value of the applied tensile stress ( $\sigma_C$ ) when the axis of the applied stress in a cubic crystal is  $[hkl]$  and the nearest slip plane is a member of  $(hkl)$ ?

In the cubic crystal system the direction  $[hkl]$  is the direction normal to the family of planes  $(hkl)$ . Consequently an  $[hkl]$  stress axis, which is normal to the  $(hkl)$  family, **cannot induce shear** on the  $(hkl)$  family of slip planes. This effect is described in the given equation for the resolved shear stress. When the direction of the applied stress and the normal to the slip plane are co-linear,  $\phi = 0$ , so  $\tau_C = 0$ .



- b. If the line direction vector ( $\xi$ ) and the Burgers vector ( $b$ ) of a dislocation are both  $[uvw]$ , the dislocation is said to be of “screw” type. Why is it called by this name?

The name “screw” dislocation comes from the offset it induces in the crystalline lattice through which it propagates. Sighting along the line direction, the atoms are arranged in a **helical** pattern, exactly like the threads of a screw.

## 5. Crystal Defects

- c. Imagine taking a Rockwell hardness test on a sample, then taking another test in exactly the same place a second time as a “double check.” How do you expect the two hardness numbers to compare? Explain.

Taking multiple hardness tests in exactly the same location **cannot** offer “cross checks” because an indentation is the result of plastic deformation, increasing the dislocation density, and therefore inducing **work hardening** during subsequent attempts at deformation by indentation. Each successive hardness number will therefore be **higher than the previous one** as the dislocation density increases under successive applications of a compressive load to the same site in the sample.

- d. When a heavily deformed material is subjected to a restorative “annealing” treatment, the defect concentration decreases. Explain.

Annealing is a **thermal treatment** conducted at elevated temperatures, on the order of  $\frac{1}{3}$  to  $\frac{1}{2}$  of the melting temperature. At these elevated temperatures, **diffusion** occurs, resulting in the motion and “intermixing” of atoms in the solid. For a heavily-deformed material, such intermixing enables atoms to move from their “defect” locations to their “normal” locations on a crystalline lattice, restoring the structure to a lower-energy equilibrium state. And annealing progresses according to the **dimensionality of the defects** induced by deformation.

Vacancies and interstitials (point defects) are removed first, during “**recovery**,” driven by a reduction in strain energy. Dislocations (line defects) are eliminated next, during “**recrystallization**,” also driven by strain energy reduction. Finally, grain boundaries (planar defects) are removed last, during “**grain growth**,” driven by surface energy reduction.