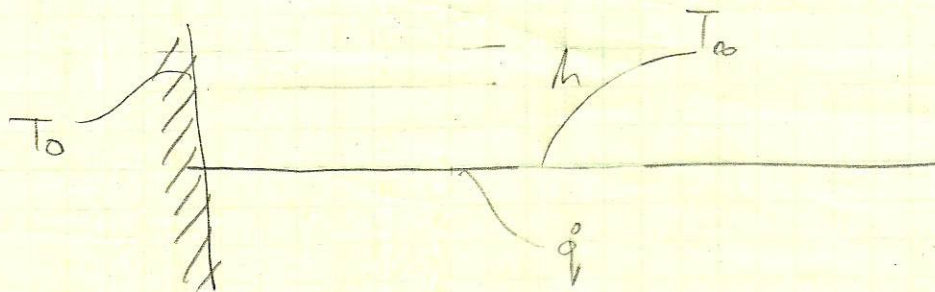
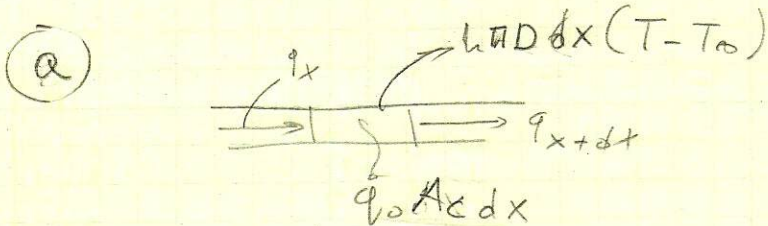


Problem 1



(a) 4 fin problem \rightarrow
 $T_{base} = \frac{q_0 \pi D^2}{4kL} + T_0$



$$q_x + \dot{q}_0 \frac{\pi D^2}{4} dx = q_{x+dx} + h \pi D (T - T_\infty) dx$$

$$-\frac{dq_x}{dx} + \dot{q}_0 \frac{\pi D^2}{4} - h \pi D (T - T_\infty) = 0$$

$$+ k \frac{\pi D^2}{4} \frac{d^2 T}{dx^2} + \dot{q}_0 \frac{\pi D^2}{4} - h \pi D (T - T_\infty) = 0 \Rightarrow$$

$$\frac{d^2 T}{dx^2} + \frac{\dot{q}_0}{k} - \frac{h \pi D}{k \frac{\pi D^2}{4}} (T - T_\infty) = 0$$

$$\frac{d^2 T}{dx^2} + \frac{\dot{q}_0}{k} - \frac{4h}{kD} (T - T_\infty) = 0 \Rightarrow$$

$$\frac{d^2 T}{dx^2} - \frac{4h}{KD} \left[T - \left(T_\infty + \frac{\dot{q}_0/k}{4h/kD} \right) \right] = 0$$

$$\frac{d^2 T}{dx^2} - \frac{4h}{KD} \left[T - \left(T_\infty + \frac{\dot{q}_0 D}{4h} \right) \right] = 0$$

$$\left. \begin{aligned} T(x=0) &= T_0 \\ \frac{dT}{dx} \Big|_{x \rightarrow \infty} &= 0 \end{aligned} \right\}$$

b) $T_0' = T_\infty + \frac{\dot{q}_0 D}{4h}$

$\theta(x) = T - T_0'$

$$\frac{d^2 \theta}{dx^2} - \frac{4h}{KD} \theta = 0$$

$$\left. \begin{aligned} \theta(0) &= T_0 - T_0' = \theta_0 \\ \frac{d\theta}{dx} \Big|_{x \rightarrow \infty} &= 0 \end{aligned} \right\}$$

$\theta(x) = \theta_0 e^{-\sqrt{\frac{4h}{KD}} x}$

$T(x) = \left(T_0 - T_\infty - \frac{\dot{q}_0 D}{4h} \right) e^{-\sqrt{\frac{4h}{KD}} x} + \left(T_\infty + \frac{\dot{q}_0 D}{4h} \right)$

d) $q_f = -k \frac{\pi D^2}{4} \frac{dT}{dx} \Big|_{x=0}$

T_{for} (C)

$q_f = +k \frac{\pi D^2}{4} \sqrt{\frac{4h}{KD}} \left(T_0 - T_\infty - \frac{\dot{q}_0 D}{4h} \right)$

Problem 2

(a) $Bi = \frac{hL}{k} = \frac{100 \cdot 0.1}{20} = 0.5$

(b) $J_1 = 0.6533$ $C_1 = 1.0701$

$\theta^* = \frac{T(x=0.1m, t) - 80}{300 - 80} = \frac{150 - 80}{300 - 80} = \frac{70}{220} = 0.32$

$\theta^* = C_1 \exp(-J_1^2 Fo) \cos(J_1 \cdot \Delta)$

$0.32 = 1.07 \cdot \exp(-0.6533^2 Fo) \underbrace{\cos(0.6533)}_{0.794}$

$\exp(-0.426 \cdot Fo) = \frac{0.32}{1.07 \cdot 0.794}$
 0.3766

$Fo = - \frac{\ln 0.3766}{0.426} \Rightarrow$

$Fo = 2.292$

$\frac{\alpha t}{L^2} = 2.292 \Rightarrow t = \frac{2.292 \cdot 0.1^2}{0.5 \cdot 10^{-5}}$

$t = 4584.5$

②

$$\theta_0^* = C_1 \exp(-s_1^2 f_0)$$

$$\theta_0^* = 1.07 \cdot \exp(-0.633^2 \cdot 2.292)$$

$$\theta_0^* = 0.427$$

$$\frac{T(x^*=0, t) - 80}{300 - 80} = 0.427 \Rightarrow$$

$$T(x=0, t) = 80 + 220 \cdot 0.427 =$$

$$T(x=0, t) = 174 \text{ } ^\circ\text{C}$$

Problem 3

$$\begin{aligned}
 \textcircled{a} \quad & \frac{d^2 T}{dx^2} = 0 \\
 & q'' = -k \frac{dT}{dx} \Big|_{x=0} \\
 & T(x=H) = 0
 \end{aligned}
 \left. \vphantom{\begin{aligned} \frac{d^2 T}{dx^2} = 0 \\ q'' = -k \frac{dT}{dx} \Big|_{x=0} \\ T(x=H) = 0 \end{aligned}} \right\}$$

$$T(x) = c_1 x + c_2$$

$$q'' = -k c_1 \Rightarrow c_1 = -\frac{q''}{k}$$

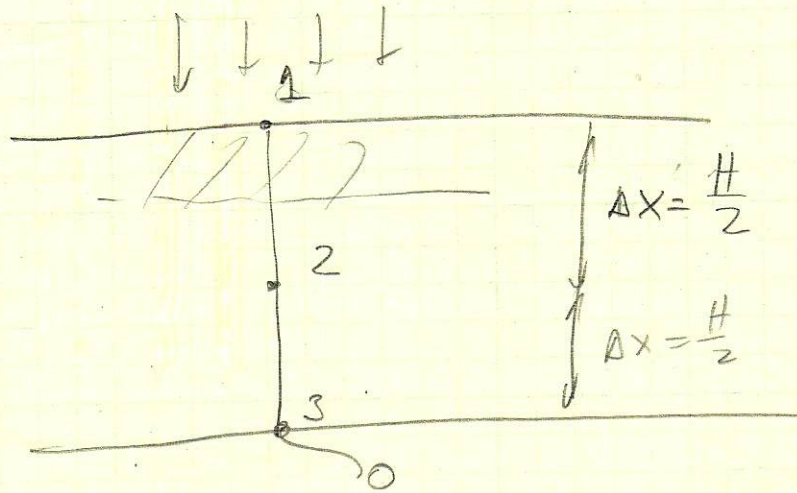
$$c_1 H + c_2 = 0 \Rightarrow c_2 = -c_1 H \Rightarrow$$

$$c_2 = +\frac{q''}{k} H$$

$$T(x) = -\frac{q''}{k} x + \frac{q''}{k} H \Rightarrow$$

$$T(x) = \frac{q''}{k} (H - x)$$

5



1

$$q'' + k \frac{T_2 - T_1}{\Delta x} = 0$$

2

$$\frac{T_1 - 2T_2 + 0}{(\Delta x)^2} = 0 \Rightarrow$$

3

$$T_1 - 2T_2 = 0$$

$$1000 + 1 \cdot \frac{T_2 - T_1}{0.05} = 0$$

$$T_1 - 2T_2 = 0$$

$$100 + 20(T_2 - T_1) = 0$$

$$\underline{T_1 = 2T_2}$$

$$100 + 20(T_2 - 2T_2) = 0$$

$$100 - 20T_2 = 0 \Rightarrow$$

$$T_2 = 5^\circ\text{C}$$

$$T_1 = 10^\circ\text{C}$$

6

$$T(0) = \frac{q'' H}{k} = \frac{100}{1} \cdot 0.1 = 10^\circ\text{C}$$

$$T(x=0.05) = \frac{q''}{k} \left(H - \frac{H}{2} \right)$$

$$T(x=0.05) = 5^\circ\text{C}$$