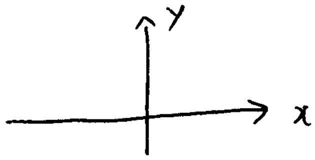
I.

(a) East $= +\hat{x}$ direction.



Jet's velocity (relative to the ground) $\vec{V}_{jet} = (500\,\text{m/s})\,\hat{x}$ —— (*)

Missle's velocity (") $\vec{V}_m = [500\,\text{m/s} + (500\,\text{m/s}^2)t]\,\hat{x}$ —(**)

$\vec{V}_{m,rel}$ = missle's velocity relative to jet

$$= \vec{V}_m - \vec{V}_{jet} = (500\,\text{m/s})t\,\hat{x} \overset{t=2}{=} \boxed{1000\,\text{m/s}\ \hat{x}}$$

$$\vec{x}_{m,rel} = \int_0^t \vec{V}_{m,rel}\,dt = \int_0^t (500\,\text{m/s})\cdot t\,\hat{x}\,dt = \frac{1}{2}(500\,\text{m/s}^2)t^2\,\hat{x}$$

$$\overset{t=2}{=} \boxed{1000\,\text{m}\ \hat{x}}$$

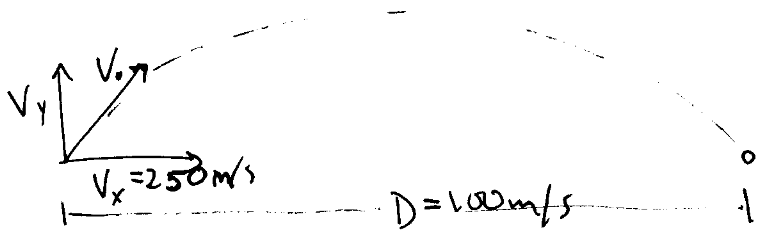
(b) From (**), we get

$$\vec{V}_m(t=2) = (500\,\text{m/s} + 1000\,\text{m/s})\hat{x} = \boxed{(1500\,\text{m/s})\,\hat{x}}$$

$$\vec{x}_m(t=2) = \int_0^2 \vec{V}_m(t)\,dt = \left[500\,\text{m/s}\cdot t + \frac{1}{2}(500\,\text{m/s}^2)t^2\right]_0^2\,\hat{x}$$

$$= \boxed{(2000\,\text{m})\,\hat{x}.}$$

2) **(15 pts)** A gun shoots bullets that leave the muzzle with a horizontal speed of 250 m/s. If a bullet is to hit a target 100 m away at the level of the muzzle, find the vertical speed of the bullet. (Neglect air resistance.)



$x = V_x t$

$\frac{D}{V_x} = t_h$

t_h is time when target is hit.

$y = V_y t - \frac{1}{2} g t^2$

$0 = V_y \cancel{t_h} - \frac{1}{2} g t_h^{\cancel{2}}$; plug t_h in when $y(t_h) = 0$

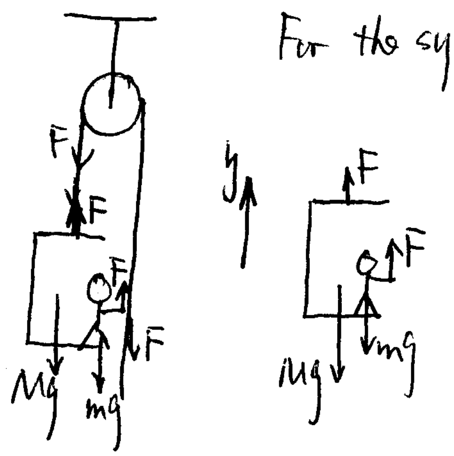
$V_y = \frac{1}{2} g t_h$

$= \frac{1}{2} \cdot g \cdot \frac{D}{V_x}$

$= \frac{1}{2} \cdot 10 \, m/s^2 \, \frac{100 \, m}{250 \, m/s}$; units check

$\boxed{V_y = 2 \, m/s}$

3.



For the system, the net force is

$$F_{net} = 2F - (m+M)g = (m+M)a$$

a) $F = \frac{1}{2}(m+M)(g+a)$

$= \frac{1}{2}\cdot(60+15)\cdot(10+0.8)$

$= 405\ N$

(If use $g = 9.8\ m/s^2$, $F = 397.5\ N$)

b) If the system goes up at a constant speed, the acceleration $\vec{a} = 0$.

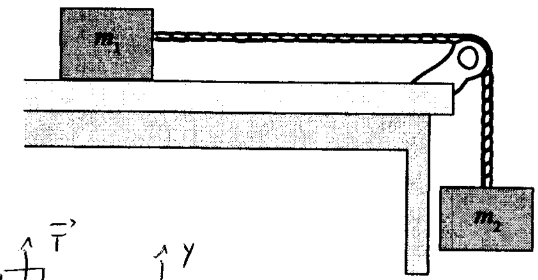
Now $F_{net} = 0$, $\Rightarrow F = \frac{1}{2}(M+m)g$

$= \frac{1}{2}\cdot(60+15)\cdot 10$

$= 375\ N$

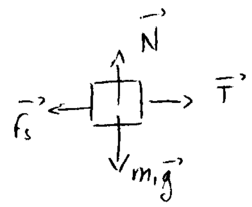
(If use $g = 9.8\ m/s^2$, $F = 367.5\ N$)

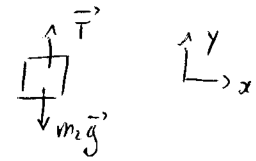
4) Consider the block of mass m_2 to be a variable mass that can be adjusted until the block of mass m_1 is on the verge of sliding on the table. (*a*) (**10 pts**) If the critical mass m_2 is 5 kg and mass m_1 is 7 kg, what is the coefficient of static friction between the table and the block? (*b*) (**10 pts**) With a slight nudge, the system accelerates at 1 m/s^2. What is the coefficient of kinetic friction between the table and the block?



a. SYSTEM IN EQUILIBRIUM

FREE BODY DIAGRAMS





$\sum_i \vec{F_i} = \vec{0}$ $\qquad$ $\sum_i \vec{F_i} = \vec{0}$

$x: \quad T - f_s = 0 \quad (1) \qquad y: \quad T - m_2 g = 0 \quad (3) \qquad$ WHERE $f_s \leq \mu_s N$

$y: \quad N - m_1 g = 0 \quad (2)$

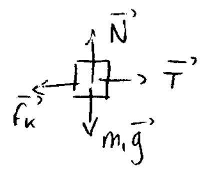
ON THE VERGE OF SLIDING $\quad f_s = \mu_s N \quad$ SO FROM (2) $\quad f_s = \mu_s m_1 g$

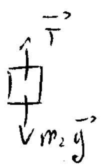
FROM (3) $\quad T = m_2 g \quad$ SO FROM (1) $\quad m_2 g - \mu_s m_1 g = 0$

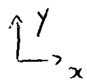
THEREFORE $\quad \mu_s = \dfrac{m_2}{m_1} = \dfrac{5}{7}$

b. SYSTEM NOT IN EQUILIBRIUM

FBD







$\sum_i \vec{F_i} = m_1 \vec{a_1} \qquad \sum_i \vec{F_i} = m_2 \vec{a_2}$

$x: \quad T - f_K = m_1 a_1 \quad (1) \qquad y: \quad T - m_2 g = -m_2 a_2 \quad (3) \qquad$ NOTE . $a_1 = a_2$ (INEXTENSIBLE CORD)

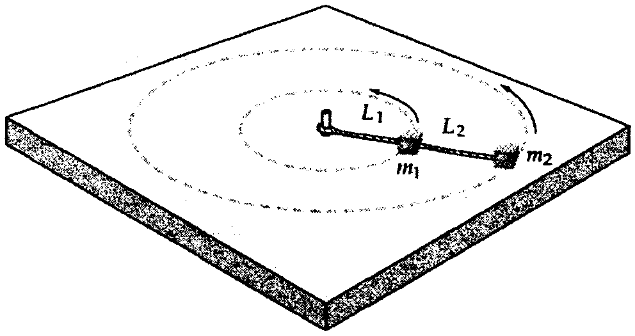
$y: \quad N - m_1 g = 0 \quad (2) \qquad\qquad$. $f_K = \mu_K N$

SINCE $\quad f_K = \mu_K N$, FROM (2) $\quad f_K = \mu_K m_1 g$

(1) - (3) $\quad -\mu_K m_1 g + m_2 g = (m_1 + m_2)\, a$

SO $\quad \mu_K = \dfrac{m_2}{m_1} - \dfrac{m_1 + m_2}{m_1} \dfrac{a}{g} = \dfrac{5}{7} - \dfrac{12}{7}\dfrac{1}{9.8} = 0.539$

5) **(15 pts)** A block of mass m_1 is attached to a cord of length L_1, which is fixed at one end. The mass moves in a horizontal circle supported by a frictionless table. A second block of mass m_2 is attached to the first by a cord of length L_2 and also moves in a circle, as shown. If the period of motion is P, find the tension in each cord.



From the period P, we can calculate the velocities of the mass m_1 and m_2,

$$P = \frac{2\pi L_1}{v_1} = \frac{2\pi(L_1+L_2)}{v_2} .$$

To have the circular motion, we need net force called centripetal force given by $F_c = \frac{mv^2}{r}$.

In this problem, the tension will supply the centripetal force.

$$\therefore T_2 = \frac{m_2 v_2^2}{r_2} = \frac{m_2 \cdot (2\pi(L_1+L_2)/P)^2}{L_1+L_2} = \underline{\underline{\frac{4\pi^2 m_2 (L_1+L_2)}{P^2}}}$$

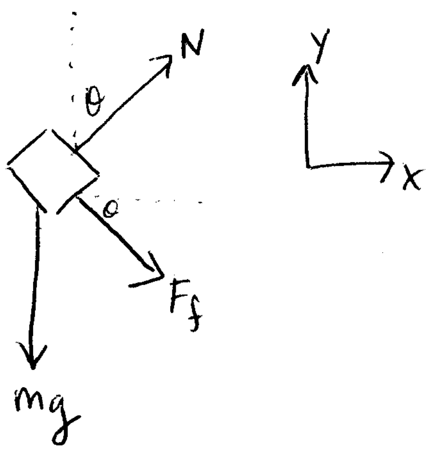
The tension for the cord 1 is the centripetal force for mass 1 and also the tension T_2.

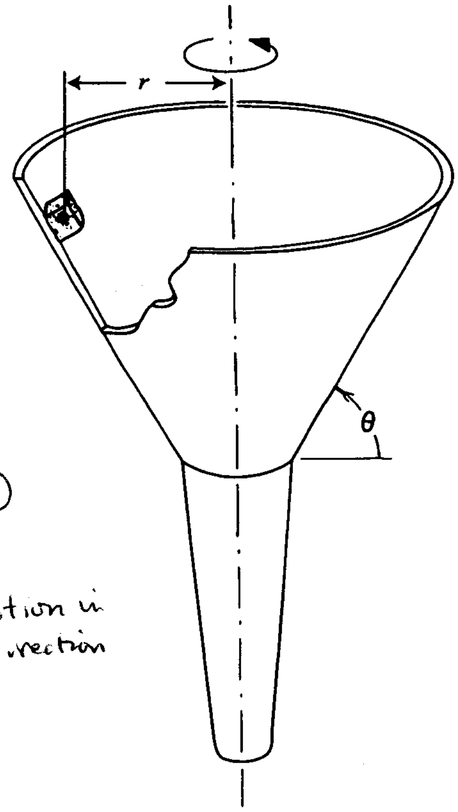
$$\therefore T_1 = T_2 + \frac{m_1 v_1^2}{r_1} = T_2 + \frac{m_1 (2\pi L_1/P)^2}{L_1}$$

$$= \underline{\underline{\frac{4\pi^2}{P^2}\left(m_1 L_1 + m_2(L_1+L_2)\right)}}$$

6) A very small cube of mass m is placed on the inside of a funnel rotating about a vertical axis at a constant rate of v revolutions per second. The wall of the funnel makes an angle θ with the horizontal. The coefficient of static friction between cube and funnel is μ_s and the center of the cube is at a distance r from the axis of rotation. Find the (*a*) (**10 pts**) largest and (*b*) (**10 pts**) smallest values of v for which the cube will not move with respect to the funnel.

a)





① $\sum F_y$: $N\cos\theta - F_f\sin\theta - mg = \cancel{m}a_y^{\,0}$

no motion in y direction

② $\sum F_x$: $N\sin\theta + F_f\cos\theta = ma_c$

$F_f = \mu_s N$

Solve equation ① for Normal force

$N(\cos\theta - \mu\sin\theta) = mg$

① $N = \dfrac{mg}{(\cos\theta - \mu\sin\theta)}$

~~Plug into~~ equation ②

② $N(\sin\theta + \mu\cos\theta) = ma_c$

Plug ① into ② Masses cancel

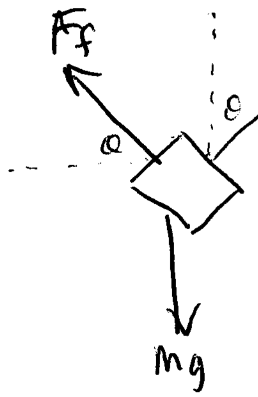
$$g\frac{(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)} = a_c$$

use $a_c = \frac{v^2}{r}$ and $v = 2\pi r \nu$

$$\frac{g(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)} = 4\pi^2 r \nu^2$$

$$\Rightarrow \quad \nu_{Max} = \sqrt{\frac{g(\sin\theta + \mu\cos\theta)}{4\pi^2 r(\cos\theta - \mu\sin\theta)}}$$

$\boxed{b}$



① ΣF_y: $N\cos\theta + F_f\sin\theta - mg = \cancel{ma_y}\ 0$

② ΣF_x: $N\sin\theta - F_f\cos\theta = ma_c$

again, solve for N, & plug into ②

① $N(\cos\theta + \mu\sin\theta) = mg$

$$N = \frac{mg}{(\cos\theta + \mu\sin\theta)}$$

② $N(\sin\theta - \mu\cos\theta) = ma_c$

① into ②

$$\frac{g(\sin\theta - \mu\cos\theta)}{(\cos\theta + \mu\sin\theta)} = a_c \qquad \text{use } a_c = \frac{v^2}{r} = \frac{2^2\pi^2\nu^2r^2}{r}$$

$$\sqrt{\frac{g(\sin\theta - \mu\cos\theta)}{4\pi^2 r(\cos\theta + \mu\sin\theta)}} = \nu_{min}$$