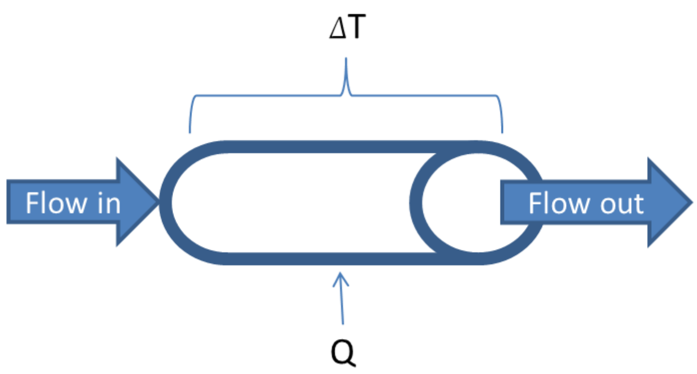
Dimensional Analysis/Model Testing

You are tasked with designing a heat exchanger around a section of piping in a synthesis plant in which temperature control will be critical to prevent bi-product formation.



In this process the temperature difference across the given length of pipe can be described as:

$\Delta T = \Delta T(\rho, \mu, V, C, Q, D)$

where density, ρ[kg/m^3], viscosity, μ[kg/(m*s)], velocity V[m/s], and heat capacity C [J/(kg*K)] are fluid properties in the real system. D [m] is the diameter of the pipe, and Q [J/s] is the total rate of heat input from the heat exchanger along entire surface area of the pipe considered. [reminder: J=kg*m^2/s^2]

a) How many dimensionless groups are there?
   7 variables – 4 dimensions = 3 dimensionless groups N1, N2, N3
b) Use Buckingham Pi theorem to find all the dimensionless groups in terms of ρ, V, C, D?

Variable	**Units**	**M**	**L**	**T**	**K**
ρ	M/L^3	1	-3	0	0
μ	M/(L*T)	1	-1	-1	0
V	L/T	0	1	-1	0
C	$L^2/(T^2K)$	0	2	-2	-1
Q	ML^2/T^3	1	2	-3	0
D	L	0	1	0	0
ΔT	K	0	0	0	-1

$N1 = \mu\rho^a V^b D^c C^d = M^0 L^0 T^0 K^0$

K: d=0

M: 1+a=0 → a=-1

L: -1-3a+b+c=0 → c=-1

T: -1-b=0 → b=-1

N1=μ/(ρVD) or (ρVD)/μ

$N2= Q\rho^a V^b D^c C^d = M^0 L^0 T^0 K^0$

K: $d=0$

M: $1+a=0 \rightarrow a=-1$

T: $-3-b=0 \rightarrow b=-3$

L: $2-3a+b+c=0 \rightarrow c=-2$

$N2= Q/(\rho D^2 V^3)$

$N3= \Delta T(\rho^a V^b D^c C^d)=M^0 L^0 T^0 K^0$

K: $1-d=0 \rightarrow d=1$

M: $a=0$

T: $-b-2d=0 \rightarrow b=-2$

L: $-3a+b+c+2d=0 \rightarrow b+c=-2 \rightarrow c=0$

$N3= (C\Delta T)/V^2$

c) You would like to run a model test on a cheaper fluid. The D and ΔT are kept the same in the model system as the real setup. There is a selection of model fluids to choose from that all have $C_{model} = 0.25C_{real}$ and $\rho_{model} = 0.5\rho_{real}$. Find the required model fluid viscosity, μ_{model} and velocity, V_{model}, along with the needed heat input from the exchanger, Q_{model}, in terms of μ_{real}, V_{real}, Q_{real} to satisfy similarity conditions.

N1, N2, N3 must be same between model and real system

First find required V from N3:

$C_{model} = 0.25C_{real}$ and $\rho_{model} = 0.5\rho_{real}$, $\Delta T_{real} = \Delta T_{model}$, $D_{real} = D_{model}$

$(C_{real}\,\Delta T_{real})/V_{real}^2 = (C_{model}\,\Delta T_{model})/V_{model}^2$

$(V_{model}/V_{real})^2 = C_{model}/C_{real}$

$V_{model}=0.5V_{real}$

To find Q, use N2:

$Q_{real}D_{real}^8/(\rho_{real}V_{real}^3) = Q_{model}D_{model}^8/(\rho_{model}V_{model}^3)$

$D_{real} = D_{model}$, and $\rho_{model} = 0.5\rho_{real}$, $V_{model}=0.5V_{real}$

$Q_{real}/Q_{model} = (\rho_{real}V_{real}^3)/(\rho_{model}V_{model}^3)$

$Q_{real}/Q_{model}=16$

To find μ for model system use N1 to satisfy similarity condition:

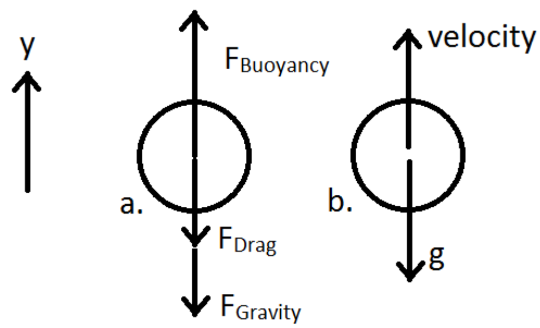
$(\rho_{real}V_{real}D_{real})/\mu_{real} = (\rho_{model}V_{model}D_{model})/\mu_{model}$

$D_{real} = D_{model}$, and $\rho_{model} = 0.5\rho_{real}$, $V_{model}=0.5V_{real}$

$\mu_{model}=0.25\mu_{real}$

(35 points) A **bubble (radius 0.05mm)** is rising through a beverage. Treat the bubble as a solid sphere and the **beverage as an infinite reservoir with all of the properties of water** at room temperature. Assume the **bubble has a density of 1.977 kg/m^3** (the density of CO_2 gas as 273K, 1 atm).

a. *Assign a coordinate system to the problem and draw vectors to represent the appropriate forces on the figure labeled "a.". Label the forces clearly, as well as where they are acting. On the figure labeled " b." draw vectors labeling the velocity of the bubble as well as the acceleration due to gravity. (3 points)*



b. *Write a force balance representing the situation diagrammed in "a." and write explicitly each term in the force balance. Derive an expression for the terminal velocity of the bubble. (7 points)*

$\Sigma F = 0 = F_{gravity} + F_{buoyancy} + F_{drag}$

$F_{gravity} = -\rho_p \pi D_p^3 |g|/6$; $F_{buoyancy} = \rho \pi D_p^3 |g|/6$; $F_{drag} = -\pi/8 * \rho * V_p^2 * D_p^2 * C_D$

$V_p^2 = 4/3 * (\rho - \rho_p)/\rho * D_p * |g|/C_D$ ➔ $\mathbf{V_p = [4/3 * (\rho - \rho_p)/\rho * D_p * |g|/C_D]^{1/2}}$

c. *Assuming Stokes flow, determine the terminal velocity of the bubble in your coordinate system. (10 points)*

Stokes flow is <u>flow about a sphere</u> at <u>Re < 1</u>.

$C_D = 24/Re$; $Re = \rho V_p D_p/\eta$

$V_p = 4/3 (\rho - \rho_p) * D_p^2 |g|/\eta * \eta/\rho V_p D_p * \rho V_p D_p/(\eta * 24) = 1/18 * (\rho - \rho_p) * D_p^2 |g|/\eta =$ **0.00544 m/s**

d. *Check whether the assumption of Stokes flow was valid for this system. Was it a good assumption? Why or why not? (5 points)*

$Re = \rho V_p D_p/\eta =$ **0.544**

Re < 1 corresponds to Stokes flow, so this was a **good assumption**.

e. *If instead of an infinite fluid reservoir, the sphere is in the center of a vertical coffee stirrer straw (cylindrical tube) of diameter 3mm, calculate the terminal velocity of the particle. You may use* $Ø = 1 + 2.10\frac{D_p}{D_c}$. *(5 points)*

Ø = 1 + 2.10* (0.1 mm / 3mm) = 1.07

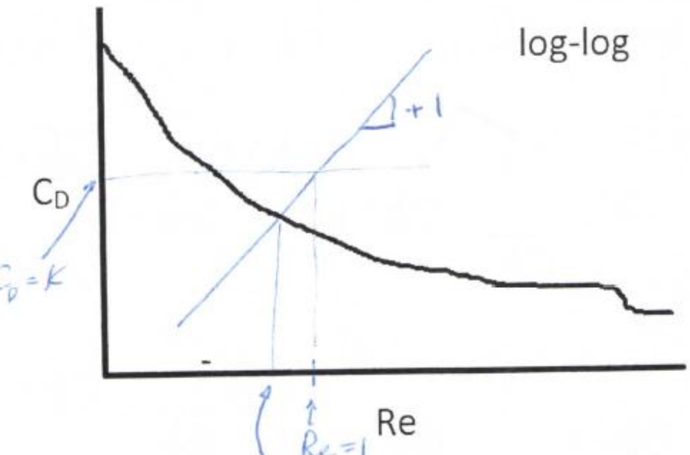
C_D = 24/Re * Ø, therefore

$V_p = 1/18 * (\rho - \rho_p) * D_p^2 |g| / \eta * 1/Ø$ = 0.00544/1.07 = **0.00508 m/s**

f. *A second CO_2 bubble rises through the liquid. Treat the liquid as infinite. If instead of the bubble diameter we know the bubble velocity, show how we can use the C_D vs. Re plot below to determine the bubble diameter. Show work to justify your method. (5 points)*

$$C_D = \frac{4}{3}\frac{(\rho_{H_2O} - \rho_{CO_2}) g D}{\rho_{H_2O} V^2}$$

$$Re = \frac{\rho_{H_2O} V D}{\eta}$$



To eliminate D:

$$\frac{C_D}{Re} = \frac{4}{3}\frac{(\rho_{H_2O} - \rho_{CO_2}) g \eta}{\rho_{H_2O}^2 V^3} = K$$

Re = Re_0

$$\Rightarrow C_D = K\,Re$$

$$\log C_D = \log K + \log Re$$

Find Re = Re_0 as shown. $D = \frac{Re_0\,\eta}{\rho_{H_2O} V}$

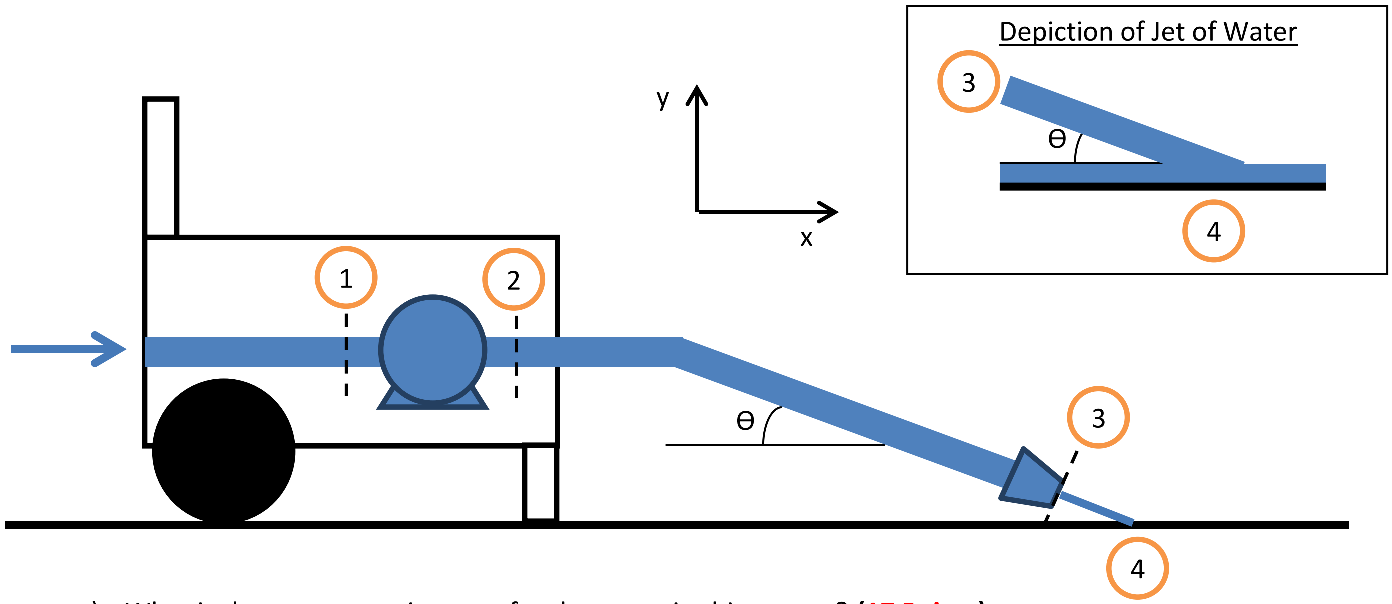
$P_1 = 3.45 \times 10^5$ Pa

$P_2 = 2.07 \times 10^7$ Pa

$Q = 1.24 \times 10^{-4}\ m^3/s$

$D_1 = D_2 = 2$ cm

$D_3 = 1.07$ mm



a) What is the power requirement for the pump in this system? (**17 Points**)

Mass Balance: $\rho<v_2>_1A_1= \rho<v_2>A_2 \quad => \quad <v_1>=<v_2>$

Engineering Bernoulli Equation:

$$\frac{\propto}{2} < v_n >_2 + gh_2 = \frac{\propto}{2} < v_n >_1 + gh_1 - \int_{P_1}^{P_2} \frac{dp}{\rho} + \delta W_S - l_v$$

$$\frac{P_2 - P_1}{\rho} = \delta W_S$$

$$\frac{2.07 \text{ x } 10^7 \text{ Pa} - 3.45 \text{ x } 10^5 \text{ Pa}}{10^3\ kg/m^3} = \delta W_S$$

$$\delta W_S = 20{,}340\ m^2/s^2$$

Calculating Power:

$$\omega = \rho \cdot Q = \left(10^3 \frac{kg}{m^3}\right) \cdot \left(1.24 \text{ x } 10^{-4} \frac{\text{m}^3}{\text{s}}\right) = 0.1262\ kg/s$$

$$W_S = \omega \cdot \delta W_S = 2567 \text{ Watts}$$

b) What is the force exerted by fluid in the y-direction at point 4? (**10 points**)

Use the control volume as shown in the depiction of the jet. In order to determine force on the ground at point 4 we'll need to do a momentum balance. The only velocity with y components is V_4, which if we use the assumption that the height is small between points 3 and 4 will be equal to V_3.

$0 = w[-V_4 \sin(\theta))] - F_{sy}$

$F_{sy} = -wV_3 \sin(\theta)$

Calculating V_3

$$Q = < V_3 > A_3 = < V_3 > \frac{\pi {D_3}^2}{4}$$

$$< V_3 > = \frac{4Q}{\pi D_3^2} = \frac{4\left(1.24 \text{ x } 10^{-4}\frac{\text{m}^3}{\text{s}}\right)}{\pi(1.07 \; x \; 10^{-3})^2} = 140.3 \; m/s$$

For a jet $<V_3> = V_3$

$$F_{sy} = -\left(0.1262\frac{kg}{s}\right)\left(140.3\frac{m}{s}\right)\sin(\theta) = -17.7\sin(\theta) \; N$$

c) Estimate the pressure felt at the surface (4), by assuming little variation in the jet stream cross-sectional area? (**8 points**)

Due to our assumptions A_3 is equal to the impact area at point 4.

$$P_{surface} = \frac{F_{sy}}{A_{impact}} = 4\frac{17.7\sin(\theta) \; N}{\pi(1.07 \; x \; 10^{-3})^2} = 19.68 \sin(\theta) \; MPa$$