Problem 1

(a) This part was not stated precisely so two solutions were accepted. In either case, we ignore thermal contraction of the glass and cork.

(i) $\frac{\Delta V}{V}$ for the air in the neck space:

$\Delta V_{neck} = -\Delta V_{wine} = 2\beta V_{0\,wine} \Delta T$ where β is the expansion coefficient for water. Then

$$\boxed{\frac{\Delta V}{V} = \frac{-2\beta V_{0\,wine}\Delta T}{\pi\left(\frac{d}{2}\right)^2 H} = .33.}$$

(ii) $\frac{\Delta V}{V}$ for wine only:

This is $\boxed{\frac{\Delta V}{V} = 2\beta(\Delta T) = -.0021.}$

(b) Observing that $V_{NS} = \pi\left(\frac{d}{2}\right)^2 H$ and assuming that d is constant,

$$\frac{V_{NS\,final}}{V_{NS\,init}} = \frac{H_f}{H_i}. \quad \text{so} \quad H_f = H_i\left(\frac{V_{NS\,init} + V_{NS\,final}}{V_{NS\,init}}\right)$$

$$\boxed{\begin{aligned} H_f &= H_i\left(1 + \left(\frac{\Delta V}{V}\right)_{neckspace}\right) \\ &= 2\text{ cm}. \end{aligned}}$$

(c) Let (V_0, V_f) (T_0, T_f) be the initial and final volumes and temperatures of the neckspace air. By the ideal gas law,

$$\frac{P_{atm} V_0}{T_0} = \frac{P_f V_f}{T_f} \quad \text{so} \quad P_f = P_{atm}\frac{T_f V_0}{T_0 V_f}.$$

To find the force the glass must exert on the cork, don't forget about the atmospheric pressure outside.

$$\boxed{F = A\,(P_{atm} - P_f) = \pi\left(\frac{d}{2}\right)^2 P_{atm}\left(1 - \frac{T_f V_0}{T_0 V_f}\right) \simeq 8.4\text{ N}.}$$

(d) The max temperature is when no neckspace remains.

$$\Delta V_{wire} = 2\beta V_{o\,wire} \Delta T = \pi \left(\frac{d}{2}\right)^2 H_o$$

$$\Delta T = \frac{\pi \left(\frac{d}{2}\right)^2 H_o}{2\beta V_{o\,wire}} \cong 15^\circ C$$

So $\boxed{T_M = T_i + \Delta T = 35^\circ C.}$

Problem 1 Rubric

This rubric is only approximate. Tests don't always fit in to these descriptions.

(a) This part was a disaster. No one really knew (myself included) what "$\Delta V / V$" meant.

- 5 Points for finding $\frac{\Delta V}{V}$ for either the neckspace or the wine (see solution)
- 3 Points for writing useful facts but not choosing a relevant definition of $\Delta V / V$.
- 2 Points for scattered calculations of various ΔV's.
- 1 Point for anything relevant

(b)
- 5 Points for a nearly Perfect answer
- 3 for something similar to $\frac{V_f}{V_i} = \frac{H_f}{H_i}$.

(c)
- 5 for a good answer (some people made differing acceptable approximations)
- 4 for everything right except forgetting to subtract atmospheric pressure
- 3 for IGL correctly used
- 2 for incorrect use of the ideal gas law
- 1 for something relevant

(d)
- 5 or 4 for nearly correct work
- 2 or 3 if it is clear that students understand the reason there is a maximum temperature
- 1 for something like "$\Delta V = \beta V_0 \Delta T$".

Problem #2

a) Sublimation of dry gas (solid → gas) 2'

$Q_{sub} = L_{ice} \cdot m_{ice} = L_{ice} \cdot (V_{ice} \cdot d_{ice})$

$= 570\ kJ/kg \times (10^{-6}\ m^3 \times 1500\ kg/m^3)$

$= 0.855\ kJ = 855\ J$ 2'

b). $R = 8.31\ J/k \cdot mol = 0.0821\ atm \cdot L / k \cdot mol$

We should calculate the number of molecules

$n_{ice} = \frac{V_{ice} \cdot d_{ice}}{M} = \frac{10^{-6} \times 1500 \times 10^3}{44} = 0.034\ mol$ 2'

$n_{gas} = \frac{P_0 V}{R \cdot T_0} = \frac{1.013 \times 10^5\ Pa/m \times 10 \times 10^{-3}\ m^3}{0.0821\ atm \cdot L/k \cdot mol \times 298\ K} = 0.409\ mol$ 2'

$\because \Sigma Q = 0$

Heat flowed out of dry ice = Heat flowed into the gas (CO_2)

$\therefore Q_{ice} + Q_{sub} + Q_{gas'} = Q_{gas}$

$\because Q_{ice} = m_{ice} C_{v\,ice} \Delta T_1$

$Q_{sub} = L_{ice} \cdot m_{ice}$

$Q_{gas'} = n_{ice} C_{v\,CO_2} \Delta T_2 = \frac{5}{2} n_{ice} R \Delta T_2$

$Q_{gas} = \frac{5}{2} n_{gas} R \Delta T_3$ 4'

$\because (1.5 \times 10^{-3}) \times (800) \times (194.5 - 183) + 855 + \frac{5}{2} \times (0.034) \times 8.31 \times (T_{eq} - 194.5)$

$= \frac{5}{2} \times (0.409) \times 8.31 \times (298 - T_{eq})$

$\therefore 13.8 + 855 + 0.70635 (T_{eq} - 194.5) = 8.496975 (298 - T_{eq})$

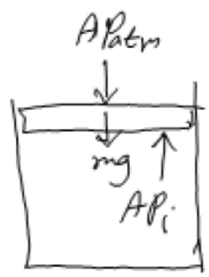
$\therefore T_{eq} = 195.7\ K$ 2'

c). Fraction $= \frac{n_{ice}}{n_{ice} + n_{gas}} = \frac{0.034}{0.034 + 0.409} = 0.0767 = 7.67\%$ 2'

d) $P_f V_f = n_f R T_f$

$P_f = \frac{n_f R T_f}{V_f} = \frac{(0.409 + 0.034) \times 0.0821 \times 195.7}{10} = 0.712\ atm$ 4'

Problem3 Solution + rubric



1

a) $\overbrace{A\cdot P_{atm} + mg = AP_i}^{1} \Rightarrow \overbrace{P_i = 102{,}280\,Pa \approx 102.3\,kPa}^{0.5}$

$\underbrace{W = \int p\,dV = P\Delta V}_{2} \underbrace{= 102{,}280 \times 1\,J \cong 102.3\,J}_{0.5}$

b) $\underbrace{Q = \Delta E + W}_{4} \underbrace{= 243\,kJ + 102.3\,kJ = 345.3\,kJ}_{1}$

c) $\underbrace{1^{st}\ \text{law: } P_i V_i = nRT_i \Rightarrow T_i = \frac{P_i V_i}{nR}}_{2} \underbrace{= 1230.8\,K}_{0.5}$

$\underbrace{P_f V_f = nRT_f \Rightarrow T_f = \frac{P_f V_f}{nR}}_{2} \underbrace{= 1353.8\,K}_{0.5}$

for one particle we have:

$$\frac{3}{2} k_B T = \left\langle \frac{1}{2} m v^2 \right\rangle$$

$$\Rightarrow \langle v^2 \rangle^{1/2} = v_{rms} = \sqrt{\frac{3 k_B T}{m}}$$

applying this to the initial & final

$$\underbrace{v_{rms,i} = \sqrt{\frac{3 k_B T_i}{m}}}_{2} = \underbrace{1009.5 \ m/s}_{0.5}$$

$$\underbrace{v_{rms,f} = \sqrt{\frac{3 k_B T_f}{m}}}_{2} = \underbrace{1058.7 \ m/s}_{0.5}$$

MIDTERM 1 - PROBLEM 4 SOLUTION

1. Part A

What condition must be satisfied at the junction between materials 1 and 2?

The rate of heat flow must be uniform throughout each component. As heat flow into the junction must equal heat flow out of the junction, the heat flow must be uniform throughout the composite material:

$$\left(\frac{dQ}{dt}\right)_1 = \left(\frac{dQ}{dt}\right)_2 \tag{1}$$

2. Part B

Determine T_J in terms of known quantities.

We now recall the equation for heat flow through a uniform material of heat conductivity k, area A, length L, and temperature difference ΔT:

$$\frac{dQ}{dt} = \frac{-kA\Delta T}{L} \tag{2}$$

Combining equations (1) and (2), we have:

$$\begin{aligned}
\frac{-k_1 A\left(T_j - T_L\right)}{x_1} &= \frac{-k_2 A\left(T_R - T_j\right)}{x_2 - x_1} \\
k_1\left(T_j - T_L\right)\left(x_2 - x_1\right) &= k_2\left(T_R - T_j\right) x_1 \\
k_1\left(x_2 - x_1\right) T_j - k_1\left(x_2 - x_1\right) T_L &= k_2 x_1 T_R - k_2 x_1 T_j \\
\left[k_1\left(x_2 - x_1\right) + k_2 x_1\right] T_j &= k_2 x_1 T_R + k_1\left(x_2 - x_1\right) T_L
\end{aligned} \tag{3}$$

We arrive at our final expression for the temperature at the junction:

$$T_j = \frac{k_2 x_1 T_R + k_1\left(x_2 - x_1\right) T_L}{k_1\left(x_2 - x_1\right) + k_2 x_1} \tag{4}$$

3. Part C

What is the rate of heat flow per surface area through region 1? Give its unit.

Applying equations (2) and (4), we have:

$$\begin{aligned}\left(\frac{dQ}{dt}\right)_1 &= \frac{-k_1 A\left(T_j - T_L\right)}{x_1} \\ &= \frac{k_1 A}{x_1}\left(T_L - T_j\right) \\ &= \frac{k_1 A}{x_1}\left(T_L - \frac{k_2 x_1 T_R + k_1\left(x_2 - x_1\right) T_L}{k_1\left(x_2 - x_1\right) + k_2 x_1}\right)\end{aligned} \tag{5}$$

The rate of heat flow per surface area through region 1 is

$$\frac{\left(\frac{dQ}{dt}\right)_1}{A} = \frac{k_1}{x_1}\left(T_L - \frac{k_2 x_1 T_R + k_1\left(x_2 - x_1\right) T_L}{k_1\left(x_2 - x_1\right) + k_2 x_1}\right) \tag{6}$$

In the SI system, this quantity has units of $\frac{J}{m^2 s}$ or $\frac{kg}{s^3}$.

It turns out that we can simplify this expression into a nicer form (this is not necessary to get full points on the problem):

$$\begin{aligned}\frac{\left(\frac{dQ}{dt}\right)_1}{A} &= \frac{k_1}{x_1}\left(\frac{k_1\left(x_2 - x_1\right) + k_2 x_1}{k_1\left(x_2 - x_1\right) + k_2 x_1} T_L - \frac{k_2 x_1 T_R + k_1\left(x_2 - x_1\right) T_L}{k_1\left(x_2 - x_1\right) + k_2 x_1}\right) \\ &= \frac{k_1}{x_1}\left(\frac{k_1\left(x_2 - x_1\right) T_L + k_2 x_1 T_L - k_2 x_1 T_R - k_1\left(x_2 - x_1\right) T_L}{k_1\left(x_2 - x_1\right) + k_2 x_1}\right) \\ &= \frac{k_1}{x_1}\left(\frac{k_2 x_1 T_L - k_2 x_1 T_R}{k_1\left(x_2 - x_1\right) + k_2 x_1}\right) \\ &= \left(\frac{k_1 k_2}{k_1\left(x_2 - x_1\right) + k_2 x_1}\right)\left(T_L - T_R\right) \\ &= -\left(\frac{\frac{k_1}{x_1} \cdot \frac{k_2}{x_2 - x_1}}{\frac{k_1}{x_1} + \frac{k_2}{x_2 - x_1}}\right) \Delta T_{tot}\end{aligned} \tag{7}$$

The expression in parentheses is the effective conductivity per length, $\frac{k_{eff}}{x_2}$.

4. Part D

Calculate k_{eff} for a temperature difference of $(T_R - T_L) = 30$ K, a total thickness $x_2 =$ 30 cm and a rate of heat flow per surface area of 10 W/m^2. Applying equation 2, we have:

$$\frac{\left(\frac{dQ}{dt}\right)_1}{A} = \frac{-k_{eff}\,(T_R - T_L)}{x_2} \tag{8}$$

As the right side is at a higher temperature than the left side, the heat flow will be negative (to the left):

$$\begin{aligned} -10\ \mathrm{W/m^2} &= \frac{-k_{eff}\cdot 30\ \mathrm{K}}{.30\ \mathrm{m}} \\ k_{eff} &= .10\ \frac{\mathrm{W}}{\mathrm{m}\cdot\mathrm{K}} \end{aligned} \tag{9}$$

Problem 5 Rubric

20 pts total

a- find W: 5 pts

- 2 pts for isochoric processes, 1 for $W = 0$, 1 for explanation
- 3 pts for adiabatic processes

b- find Q: 5 pts

- 2 pts for adiabatic processes, 1 for $Q = 0$, 1 for explanation
- 3 pts for isochoric processes

c- find ΔS: 5 pts

- 2 pts for adiabatic processes, 1 for $\Delta S = 0$, 1 for explanation
- 3 pts for isochoric processes

d- find e 5 pts

- 2 pts for obtaining expression in terms of W's and Q's
- 2 pts for using $PV^\gamma = constant$
- 1 pt for simplifying final expression

Problem 5

- state a: pressure P_a and volume V_a
- state b: pressure P_b and volume V_b
- state c: pressure P_c and volume $V_c = V_b$
- state d: pressure P_d and volume $V_d = V_a$
- degrees of freedom: $d = 5 \quad \therefore \gamma = \frac{d+2}{d} = 7/5$

a- find W

- $(a \to b)$ and $(c \to d)$ are adiabatic

$$\therefore Q = 0 \qquad \Delta E = Q - W = -W$$

$$W = -\Delta E = \frac{-d}{2} nR\Delta T = \frac{-d}{2}\Delta(PV) = \frac{-d}{2}(P_f V_f - P_0 V_0) = \frac{5}{2}(P_0 V_0 - P_f V_f)$$

$$W_{ab} = \frac{5}{2}(P_a V_a - P_b V_b) \qquad W_{cd} = \frac{5}{2}(P_c V_b - P_d V_a)$$

- $(b \to c)$ and $(d \to a)$ are isochoric

$$\therefore dV = 0 \qquad W_{bc} = 0 \qquad W_{da} = 0$$

b- find Q

- $(a \to b)$ and $(c \to d)$ are adiabatic, by definition

$$Q_{ab} = 0 \qquad Q_{cd} = 0$$

- $(b \to c)$ and $(d \to a)$

$$Q = nC_v \Delta T \ \mathit{for\ constant}\ V$$

$$Q = \frac{d}{2} nR\Delta T = \frac{d}{2} V\Delta P = \frac{5}{2} V(P_f - P_0)$$

$$Q_{bc} = \frac{5}{2} V_b (P_c - P_b) \qquad Q_{da} = \frac{5}{2} V_a (P_a - P_d)$$

c- find ΔS

- $(a \to b)$ and $(c \to d)$

$$Q = 0 \ \therefore \Delta S = \int \frac{dQ}{T} = 0$$

$$\Delta S_{ab} = 0 \qquad \Delta S_{cd} = 0$$

- $(b \to c)$ and $(d \to a)$

$$\Delta S = \int \frac{dQ}{T} = \int_{T_0}^{T_f} \frac{(d/2)nRdT}{T} = \frac{d}{2} nR \ln\left(\frac{T_f}{T_0}\right)$$

$$\mathit{isobaric} \ \therefore T_f/T_0 = P_f/P_0 \quad \Delta S = \frac{5}{2} nR \ln\left(\frac{P_f}{P_0}\right)$$

$$\Delta S_{bc} = \frac{5}{2} nR \ln\left(\frac{P_c}{P_b}\right) \qquad \Delta S_{da} = \frac{5}{2} nR \ln\left(\frac{P_a}{P_d}\right)$$

d- find e

$$e = \frac{W_{net}}{Q_{in}} = \frac{W_{ab} + W_{cd}}{Q_{bc}} = \frac{(5/2)(P_a V_a - P_b V_b) + (5/2)(P_c V_b - P_d V_a)}{(5/2) V_b (P_c - P_b)} = \frac{-V_a(P_d - P_a) + V_b(P_c - P_b)}{V_b(P_c - P_b)}$$

For the adiabatic processes

$$P_a V_a^\gamma = P_b V_b^\gamma \to P_a = P_b V_b^\gamma / V_a^\gamma$$

$$P_c V_c^\gamma = P_d V_d^\gamma \to P_d = P_c V_c^\gamma / V_d^\gamma = P_c V_b^\gamma / V_a^\gamma$$

Substitute expressions for P_a and P_d in efficiency expression above

$$e = \frac{-V_a(V_b^\gamma / V_a^\gamma)(P_c - P_b) + V_b(P_c - P_b)}{V_b(P_c - P_b)}$$

Cancel out $(P_c - P_b)$ and simplify

$$e = \frac{V_b - V_b^\gamma V_a^{1-\gamma}}{V_b} = 1 - \left(\frac{V_b}{V_a}\right)^{\gamma - 1} = 1 - \left(\frac{V_b}{V_a}\right)^{2/5}$$