

PHYSICS 7B, Section 1 - Fall 2012
Midterm 1, C. Bordel
Monday, September 24, 2012
7pm-9pm

Physical constants

$$k_B = 1.38 \times 10^{-23} \text{ J/K} ; R = 8.31 \text{ J/K.mol.}$$

Conversions

$$1 \text{ L} = 10^{-3} \text{ m}^3 ; T(\text{K}) = T(^{\circ}\text{C}) + 273 ; 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

Problem 1 - Thermal expansion (20 pts)

Wine bottles are never completely filled. A small volume is left in the cylindrical neck of the bottle of diameter d to allow for the thermal expansion of wine. H is the distance between the surface of the liquid and the bottom of the cork. Typically, $H=1.5 \text{ cm}$ for a 750 mL bottle of wine filled at 20°C , and $d=2 \text{ cm}$.

The volumetric coefficients of thermal expansion of glass and water are respectively $27 \times 10^{-6}/^{\circ}\text{C}$ and $210 \times 10^{-6}/^{\circ}\text{C}$.

- a- Assuming that the coefficient of thermal expansion of wine is about double that of water, and that of the cork is of the same order of magnitude of that of glass, make an approximate calculation of the relative volume expansion $\Delta V/V$ when the wine is cooled from 20°C to 15°C .
- b- What is the distance H at $T=15^{\circ}\text{C}$?
- c- If the wine was bottled at 20°C and then chilled to 15°C , what is the approximate force holding the cork in, assuming no friction against the glass?
- d- Assuming that air is infinitely compressible, what is the maximum temperature T_m allowing for the wine's thermal expansion from 20°C ?

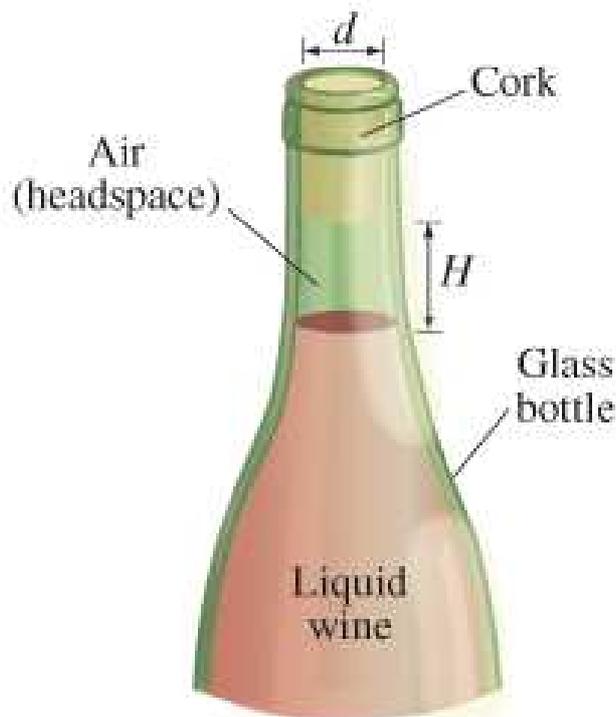


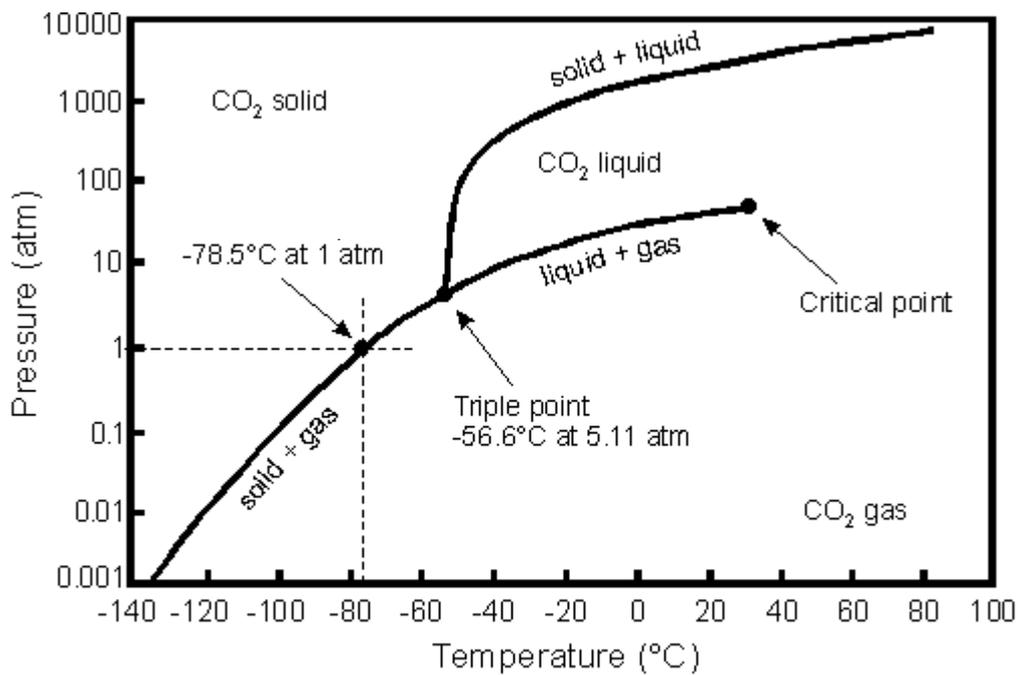
Figure 1

Problem 2 - Phase change (20 pts)

Gaseous carbon dioxide, considered as an ideal gas, is placed in an insulated vessel where the thermodynamic conditions are $P = 1 \text{ atm}$, $V = 10 \text{ L}$, $T = 25 \text{ }^\circ\text{C}$. Note that CO_2 molecules are linear and assume the system is not hot enough to have appreciable vibrational kinetic energy.

A cube of dry ice of volume $V_{ice} = 1 \text{ cm}^3$ with temperature $T_{ice} = -90 \text{ }^\circ\text{C}$ is added to the vessel in which the conditions are as above. The density of dry ice is $d_{ice} = 1500 \text{ kg/m}^3$. The phase change temperature is to be read on Fig.2. The latent heat of the transformation is $L_{ice} = 570 \text{ kJ/kg}$ and the constant volume heat capacity is $C_{v,ice} = 0.8 \text{ kJ/kg.K}$. The molar mass of CO_2 is 44 g/mol .

- According to the phase diagram shown in Fig.2, what is the phase transformation that will occur in the vessel? What is the amount of heat involved in this phase change?
- If the volume of the vessel is held constant as the ice transforms, determine the equilibrium temperature.
- What fraction of carbon dioxide molecules within the vessel originate from the dry ice cube?
- What is the final pressure within the vessel?



Pressure-Temperature phase diagram for CO_2 .

Figure 2

Problem 3 – First law (20 pts)

100 moles of air, considered as an ideal diatomic gas, is contained in a vertical piston-cylinder assembly of initial volume $V_i = 10 \text{ m}^3$ fitted with an electrical heater, as sketched in Fig.3. Atmospheric pressure is exerted on the top of the piston, which has a mass of $m = 100 \text{ kg}$ and a surface area $A = 1 \text{ m}^2$. Electrical current passes through the resistor and the volume of the air slowly increases by $\Delta V = 1 \text{ m}^3$ while the pressure remains constant. During this process the total energy of the air increases by $\Delta E_{int} = 243 \text{ kJ}$, and we assume the system is not hot enough to have appreciable vibrational kinetic energy. The air and piston are at rest initially and finally. The piston-cylinder material is a ceramic composite providing a good thermal insulation.

- a- Using a force diagram, determine the initial pressure P_i in the cylinder, and determine the work done by the gas during this thermodynamic process.
- b- What is the total heat transferred to the gas by the heater?
- c- What are the initial and final temperatures T_i and T_f ?
- d- What is the rms velocity of the gas molecules at the initial and final temperatures? Use $M=5 \times 10^{-26} \text{ kg}$ for the molecular mass.

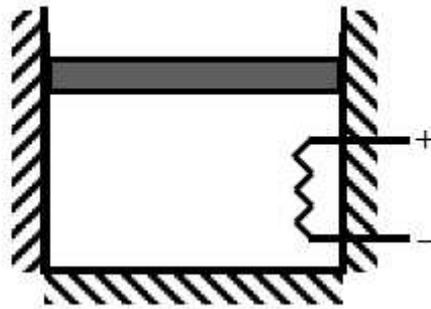


Figure 3

Problem 4 – Conductive heat transfer (20 pts)

Two slabs of different materials form a junction depicted in Fig. 4, with a constant surface area A . The temperatures within the materials have reached equilibrium conditions. The boundaries at $x=0$ and $x=x_2$ are held at constant temperatures, T_L and T_R respectively. The two slabs are denoted by regions I and II, and the associated thermal conductivities are k_I and k_{II} , assumed constant within each material.

- a- What condition must be satisfied at the junction between materials I and II?
- b- Determine T_j in terms of known quantities.
- c- What is the rate of heat flow per surface area through region I? Give its unit.
- d- Now we consider this bilayer as a single layer of effective thermal conductivity k_{eff} . Calculate k_{eff} for a temperature difference $T_R - T_L = 30 \text{ K}$, a total thickness $x_2 = 30 \text{ cm}$ and a rate of heat flow per surface area of 10 W/m^2 .

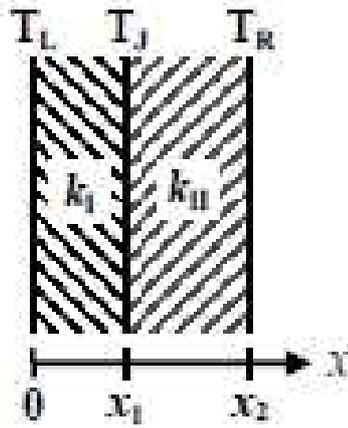


Figure 4

Problem 5 - Heat engine - Otto cycle (20 pts)

The operation of an automobile internal combustion engine can be approximated by a reversible cycle known as "Otto" cycle, which involves two adiabatic paths and two isochoric paths, as shown in Fig.5. Assume we are using n moles of an ideal diatomic gas as the working substance and assume the system is not hot enough to have appreciable vibrational kinetic energy .

- a- Express the work done by the gas along each branch of the cycle in terms of pressures and volumes.
- b- What are the heat flows for each of the four processes in this cycle in terms of pressures and volumes?
- c- What is the entropy change of the gas along each branch of the cycle?
- d- Give the efficiency of this cycle as a function of V_a and V_b only.

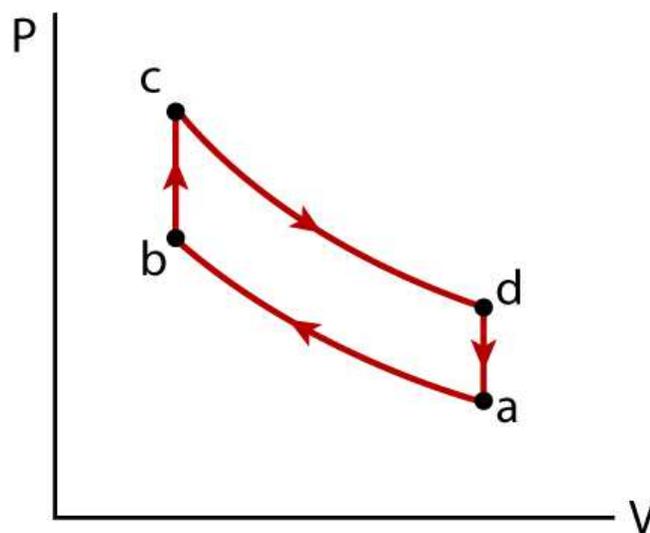


Figure 5