Department of Mechanical Engineering
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# First Midterm Examination
# Closed Books and Closed Notes

## Question 1
*Vehicle Dynamics* (25 POINTS)

An engineer is commissioned to design a track profile to test the suspension and driving dynamics of a car. The road has the following vertical profile:

$$y = f(x) \text{ where } f(x) = A \sin\left(\frac{\pi x}{L}\right), \tag{1}$$

where $A$ and $L$ are constants. The car is modeled as a particle of mass $m$ which is subject to a normal force $\mathbf{N}$, a traction force $\mathbf{F}_T = F_T\mathbf{e}_t$, a drag force $-mC_d \|\mathbf{v}\|^2 \mathbf{v}$ and a gravitational force $-mg\mathbf{E}_y$.

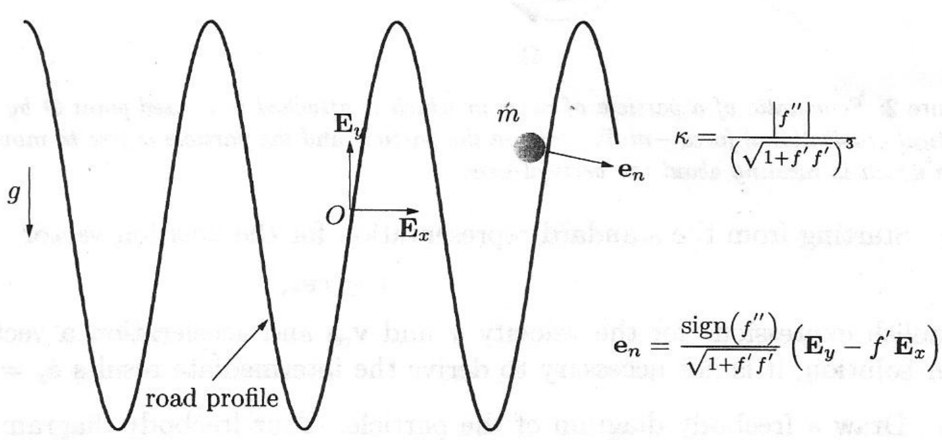


Figure 1: *Schematic of a particle of mass m moving on a road.*

**(a)** *(8 Points)* Starting from the following representation for the position vector of the particle,

$$\mathbf{r} = x\mathbf{E}_x + f(x)\mathbf{E}_y, \tag{2}$$

derive expressions for the speed $v$, velocity vector $\mathbf{v}$, and acceleration vector $\mathbf{a}$ of the particle. Assuming $\dot{x} > 0$, for which locations on the road are $v = \dot{x}$?

**(b)** *(5 Points)* Draw a freebody diagram of the particle.

**(c)** *(6 Points)* Assuming that the car is moving at a constant speed $v_0$ on the given road profile with $\dot{x} > 0$, show that the traction force and the normal force acting on the car are, respectively,

$$\mathbf{F}_T = \left(mC_d v_0^3 + mg??\right)\mathbf{e}_t, \qquad \mathbf{N} = \left(mv_0^2\kappa + mg???\right)\mathbf{e}_n. \tag{3}$$

For full credit, you need to give correct expressions for the terms denoted by ?? and ???.

**(d)** *(6 Points)* For which locations on the track are $\mathbf{F}_T$ maximized and minimized? Give a physical interpretation of your solutions. Hint: $\frac{d}{dx}\frac{mgf'}{\sqrt{1+f'f'}} = \frac{mgf''}{\left(\sqrt{1+f'f'}\right)^3}$.

## Question 2

*A Particle in a Rotating Tube* (25 POINTS)

As shown in Figure 2, a particle of mass $m$ is attached to a fixed point $O$ by a linearly elastic spring. The spring has a stiffness $K$ and an unstretched length $L$. The particle is free to move inside a rough tube which is rotating about a vertical axis with a speed $\Omega(t)$.

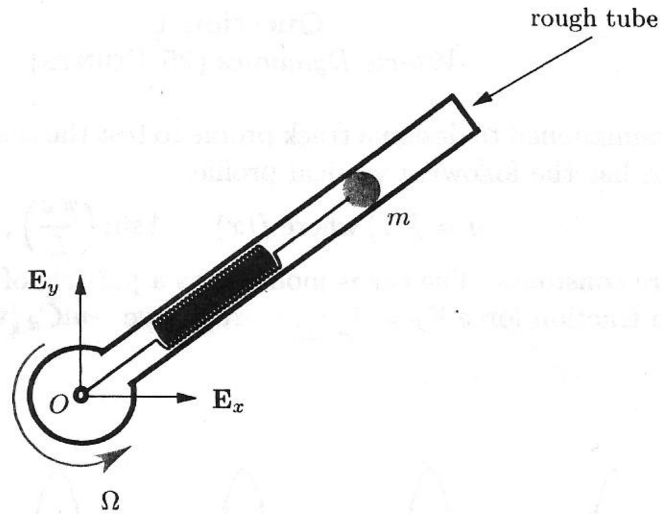


Figure 2: *Schematic of a particle of mass m which is attached to a fixed point O by a linearly elastic spring. A vertical gravitational force $-mg\mathbf{E}_z$ acts on the particle and the particle is free to move on the inside of the rough tube which is rotating about the vertical axis.*

**(a)** Starting from the standard representation for the position vector

$$\mathbf{r} = r\mathbf{e}_r, \tag{4}$$

establish expressions for the velocity $\mathbf{v}$ and $\mathbf{v}_{\text{rel}}$ and acceleration $\mathbf{a}$ vectors of the particle. In your solution, it is not necessary to derive the intermediate results $\dot{\mathbf{e}}_r = \dot{\theta}\mathbf{e}_\theta$ and $\dot{\mathbf{e}}_\theta = -\dot{\theta}\mathbf{e}_r$.

**(b)** Draw a freebody diagram of the particle. Your freebody diagram should include a clear expression for the spring force and accommodate both the static and dynamic friction cases.

**(c)** Suppose that the particle is moving relative to the tube. Show that the differential equation governing the motion of the particle is

$$m\left(\ddot{r} - r\Omega^2\right) = ?? - \mu_k \|\mathbf{N}\| ???. \tag{5}$$

For full credit show how $\mathbf{N}$ in (5) can be determined and provide expressions the terms denoted by ?? and ???

**(d)** Suppose that the particle is stationary relative to the tube: $r = r_0$. Show that it is possible for the particle to remain in this state provided

$$\left|\frac{K}{m}(r_0 - L) - r_0\Omega^2\right| \le \mu_s\sqrt{g^2 + r_0^2\dot{\Omega}^2}. \tag{6}$$

**(e)** Henceforth restrict attention to the case where the tube is smooth and $\Omega = \Omega_0$ is constant. Suppose the particle is placed in the tube with $r = r_0$ and $\dot{r} = 0$. Show that in the subsequent motion

$$\dot{r}^2 = \Omega_0^2\left(r^2 - r_0^2\right) - \frac{K}{m}\left((r - L)^2 - (r_0 - L)^2\right). \tag{7}$$

Hint: one approach to solving this problem is to use the identity $vdv = adr$.