IEOR 165 MT2 Spring 2012

1. chickens $n=525$

$\hat{p}=0.83$

$p \leftarrow \hat{p}$

Assumptions for CLT

1. SRS
2. Samples are iid
3. n is small relative to size of population
4. $np \geq 10$ $n(1-p) \geq 10$

(a) 95% CI for $\hat{p}$

$\hat{p} \pm z_{0.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \Rightarrow [0.7978, 0.8621]$

We are 95% confident that the ~~true population~~ true proportion of checked infected is captured by [0.7978, 0.8621]

(b) The statement of the spokesperson is invalid because

(1) #3 assumption for CLT ⇒ against it

(2) Data in sample should be representative

2

	M	W
n:	50	50
$\bar{X}_m$:	19.39	17.91
S_m	2.52	3.39

(a) $n>30$, 95% CI for male

$\bar{X} \pm z_{0.025}\sqrt{\frac{s^2}{n}}$

$z: [18.69149, 20.088]$

$t_{49}: [18.67382, 20.1048]$

either is okay!

95% CI for female

$z: [16.9703, 18.8496]$

(b) $H_0: \mu_M - \mu_W = 0$

$H_1: \mu_M - \mu_W \neq 0$

$S_d = \sqrt{\frac{S_M^2}{n_M} + \frac{S_W^2}{n_W}} = 0.5973$

$z = \frac{(\bar{X}_M - \bar{X}_W) - 0}{S_d} = 2.47$

P-value $= 2 * Pr\{z > 2.47\} = 0.0132$

Reject H_0: There is a significant difference in the performances of males & females.

(c) 95% MSE $= 1.96 * Sd = 1.1708$

$[0.309, 2.651]$

3. 1996 20%

2000 p $\leftarrow$ $n=1100$ $\hat{p}=0.25$

H_0: $p=0.2$

H_1: $p>0.2$

Assumptions:

1. SRS
2. iid
3. small enough
4. large $np\geq 10$ & $n(1-p)$

$S_d=\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}=0.01305$

95% CI: $[0.224, 0.275]$

P-Value $=\Pr\left\{Z>\frac{0.25-0.2}{0.01305}\right\}=\Pr\{Z>3.829\}\approx 0$

Reject $H_0 \Rightarrow$ Yes, there is evidence for grade inflation.

4.

June	54	49	68	66	62	62	$\leftarrow$ group 2
Aug.	50	65	74	64	68	72	$\leftarrow$ group 1

Paired t-test

d = score in Aug. − score in June

H_0: $d=0$

H_1: $d>0$

$\bar{d}=5.35$ $sd(d)=3.04$

$T_s=\frac{\bar{d}-0}{sd(d)}=1.7541$

P-Value $=\Pr\{t_s>1.7541\}=0.069$

95% CI for d: $(-7.56, 8.06)$

fail to reject H_0: There is no sufficient evidence to say the program is worthwhile.

5. n · size = 6 < 30

$\bar{X} = 28.8 \quad S_x = 0.4$

$S_{\bar{x}} = \frac{S_x}{\sqrt{6}} = 0.1632$

95% CI: $\bar{X} \pm t_{5, 0.025} S_{\bar{x}}$

$= [28.38,\ 29.219]$

Assumptions: ① 6 bags from SRS

② 6 bags iid from normal distribution

$H_0: \mu = 28.3$

$H_1: \mu \neq 28.3$

$t = \frac{\bar{X} - 28.3}{S_{\bar{x}}} = 3.0618$

P-Value = $2 \cdot P\{t_5 > 3.0618\} = 0.028$

Reject H_0: There is significant evidence to show that the true weight of this kind of bags is different from 28.8 grams.

Assumption: ① SRS

② 6 bags are iid from $N(28.3, \sigma^2)$

6.

	A	B
n	112	108
Sx	84	66

$p_i = \frac{S_i}{n_i}$

$H_0: p_1 = p_2$

$H_1: p_1 \neq p_2$

pooled: $\hat{p}_{pooled} = \frac{S_1 + S_2}{n_1 + n_2} = 0.6818$

$Sd = \sqrt{\hat{p}_{pooled}(1 - \hat{p}_{pooled})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$

$z = \frac{\hat{p}_1 - \hat{p}_2 - 0}{Sd} = 2.211$

P-Value = $2 \cdot P\{Z > 2.211\} = 0.027$

Assumptions:

1. SRS
2. n_1 & n_2 are small enough relative to populations
3. $n_1 p_1 \geq 10 \quad n_1(1-p_1) \geq 10$

   $n_2 p_2 \geq 10 \quad n_2(1-p_2) \geq 10$
4. two groups are iid of each other
5. samples in both groups are iid

(b) 95% CI

$\Rightarrow (0.01688,\ 0.26089)$

$SE(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} = 0.062$