Department of Physics
University of California, Berkeley

Mid-term Examination 1
Physics 7B, Sections 2 and 3

6:30 pm - 8:30 pm, October 4, 2005

Name:______________________________________

SID No:______________________________________

Discussion Section:______________________________

Name of TA:___________________________________

Answer all five problems. Write clearly and explain your work. Partial credit will be given for incomplete solutions provided your logic is reasonable and clear. Cross out any parts that you don't want to be graded. Enclose your answers with boxes. **Express all numerical answers in SI units**. Answers with no explanation or disconnected comments will not be credited. If you obtain an answer that is questionable, explain why you think it is wrong.

Constants and Conversion factors

Avogadro number, N_A	6.022×10^{23}
Universal gas constant, R	$8.315\ J{\cdot}mol^{-1}{\cdot}K^{-1} = 1.99\ cal{\cdot}mol^{-1}{\cdot}K^{-1}$
Boltzmann constant, k	$1.381 \times 10^{-23}\ J{\cdot}K^{-1}$
Stefan-Boltzmann constant, σ	$5.67 \times 10^{-8}\ W{\cdot}m^{-2}{\cdot}K^{-4}$
Acceleration due to gravity, g	$9.8\ m{\cdot}s^{-2}$
Specific heat of water	$1\ kcal{\cdot}kg^{-1}{\cdot}{}^{\circ}C^{-1}$
Heat of fusion of water	$80\ kcal{\cdot}kg^{-1}$
1 atm	$1.013 \times 10^{5}\ N{\cdot}m^{-2}$
1 kcal	4.18×10^{3} J
1 BTU	1055 J
1 hp	746 W

R r

1. A hollow plastic sphere at 20 °C is found to have an inner radius of 1.925 cm and an outer radius of 3.142 cm at 20 °C. At this temperature, the density of the plastic is 1030 $kg{\cdot}m^{-3}$. Then the sphere is dropped into a tank of hot water maintained at 100 °C and no water is leaked into the sphere.

(a) [3 points] What is the overall density of the sphere at 20 °C? Ignore the mass of air inside the sphere.

$$V_{plastic} = \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left((0.03142\ m)^3 - (0.01925\ m)^3\right)$$

$$= 1.0005 \times 10^{-4}\ m^3.$$

$$\rho_{total} = \frac{M_{plastic}}{V_{total}} = \frac{\rho_{plastic} V_{plastic}}{\frac{4}{3}\pi R^3} = \frac{(1030\ kg\ m^{-3})(1.0005\times10^{-4}\ m^3)}{\frac{4}{3}\pi\ (0.03142\ m)^3}$$

$$= \frac{1.031\times10^{-1}\ kg}{1.299\times10^{-4}\ m^3} = 793.14\ kg/m^3.$$

(b) [12 points] After the sphere has reached thermal equilibrium with the hot water, what are the radii of the sphere? The average coefficient of thermal expansion of the plastic is 1 $\times10^{-5}$ per °C.

$$\Delta R = \alpha R \Delta T = (10^{-5}\ K^{-1})(0.03142\ m)(80\ K) = 2.5136\times10^{-5}\ m$$

$$\Delta T = 100\ °C - 20\ °C = 80\ °C = 80\ K.$$

$$R' = R + \Delta R = 0.03142\ m + 2.5136\times10^{-5}\ m = 0.03145\ m.$$

$$\Delta r = \alpha r \Delta T = (10^{-5}\ K^{-1})(0.01925\ m)(80\ K) = 1.54\times10^{-5}\ m.$$

$$r' = r + \Delta r = 0.01925\ m + 1.54\times10^{-5}\ m = 0.01927\ m.$$

(c) [3 points] What is the overall density of the sphere after its temperature is at 100 °C? Ignore the mass of air inside the sphere.

$$\rho'_{total} = \frac{M_{plastic}}{V'_{total}} = \frac{1.031 \times 10^{-1}\ kg}{\frac{4}{3}\pi R'^3} = \frac{0.1031\ kg}{\frac{4}{3}\pi (0.03145\ m)^3}$$

$$= \frac{0.1031\ kg}{1.309 \times 10^{-4}\ m^3} = \cancel{787.48}\ kg/m^3.$$

791.23

$M_{plastic}$ does not change.

(d) [2 points] Assuming the density of water at 100 °C is about 998 kg·m^{-3}, does the sphere sink or float in the water initially, and after it reaches thermal equilibrium?

ρ_{total} and ρ'_{total} are both less than the density of water.

The sphere floats in both cases.

2 (a) [5 points] The average surface temperature of Mars is -63 °C. What are the mean speeds of a hydrogen molecule and an oxygen molecule near the surface of Mars? Given the escape speed of Mars is 5.0 km/s, can the oxygen molecule eventually leave Mars?

$$[J] = kg\,\frac{m^2}{s^2}$$

Equipartition says $\langle E_{translational} \rangle = \langle \frac{1}{2} m v_{rms}^2 \rangle = \frac{3}{2} kT$

$$\therefore\ v_{rms} = \sqrt{\frac{3kT}{m}} = \left(\frac{3 \cdot 1.381 \times 10^{-3}\,J/K \cdot (273-63)\,K}{2 \cdot 1.66 \times 10^{-27}\,kg} \right)^{1/2} = 1620\ m/s \text{ for } H_2$$

$$= \left(\frac{3 \cdot 1.381 \times 10^{-3}\,J/K \cdot (273-63)\,K}{32 \cdot 1.66 \times 10^{-27}\,kg} \right)^{1/2} = 405\ m/s \text{ for } O_2.$$

2 and 3 are the masses (in amu) of H_2 and O_2. At this speed, the oxygen cannot escape from Mars. However, by colliding with other particles, it may find itself at the high velocity tail of the Maxwell velocity distribution, allowing it to escape. *

(b) [5 points] In the LEP particle accelerator, the electrons go around a circular path of circumference 27 km in a chamber of 10^{-12} mm Hg pressure and 300 K temperature. What is the number density of the residual gas in the chamber? What would be the mean free path of the gas molecules under these conditions if the cross section is 2×10^{-15} cm^2?

$pV = NkT$ number density $= n = \frac{N}{V} = \frac{P}{kT}$

$$n = \frac{P}{kT} = \frac{10^{-12}\ mmHg}{1.381 \times 10^{-23} \cdot 300\,K} \ \frac{1\ atm}{760\ mmHg} \ \frac{101.3 \times 10^3\ Pa}{1\ atm} = 3.2 \times 10^{11}\ m^{-3}.$$

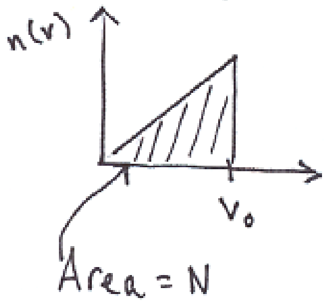
$$\text{Mean free path} = \frac{1}{n\sigma} = \frac{1}{3.2 \times 10^{11}\ m^{-3}} \ \frac{1}{2 \times 10^{-19}\ m^2} = 15{,}500\ km$$

This answer can be obtained by dimensional analysis. More exact answers differ by a factor of order unity.

* This, evidently, is what has happened, as Mars has a very thin ~~atmosphere~~ atmosphere.

(c) Suppose the speed distribution of N particles is given by $n(v) = Cv$ for $0 < v < v_0$ and $n(v) = 0$ for $v > v_0$.

(i) [4 points] Determine the normalization constant C as a function of N and v_0.



n(v) dv describes the number of particles with speeds between v and v + dv. The total number is

$$N = \int_0^\infty n(v)\,dv = \int_0^{v_0} Cv\,dv = C\frac{v_0^2}{2}$$

$$\Rightarrow \quad C = \frac{2N}{v_0^2}$$

(ii) [6 points] Determine the mean speed and the root-mean-square speed of the particles.

$$\text{Mean speed} = \langle v \rangle = \frac{\int_0^{v_0} v\,n(v)\,dv}{N} = \frac{\frac{2N}{v_0}}{N}\int_0^{v_0} v^2\,dv$$

$$= \frac{2}{v_0}\,\frac{v_0^3}{3} = \frac{2}{3}v_0.$$

$$\text{Root mean squared speed} = \sqrt{\langle v^2 \rangle}$$

$$\langle v^2 \rangle = \frac{1}{N}\int_0^{v_0} v^2 n(v)\,dv = \frac{2N/v_0^2}{N}\int_0^{v_0} v^3\,dv = \frac{2}{v_0^2}\,\frac{v_0^4}{4} = \frac{v_0^2}{2}.$$

$$\sqrt{\langle v^2 \rangle} = \frac{1}{\sqrt{2}}v_0.$$

3. A thin-wall cylindrical pipe is used to carry hot water at a temperature of 90 °C. The diameter of the pipe is 2.54 cm and it is wrapped around with a 2.54-cm-thick insulation. A section of the pipe, 5 m long, passes through a room at a temperature of 10 °C.

(a) [12 points] What is the rate of heat loss through the insulation? The thermal conductivity of the insulation is 0.05 $W \cdot m^{-1} K^{-1}$.

For a bar, $H = \frac{dQ}{dt} = -k \frac{A \Delta T}{l}$

For the cylinder, the area varies: $A(r) = 2\pi r L$

If we take a thin slice of the cylinder, the area won't change much and we can apply the bar formula:

$$H = -k \frac{2\pi r L \, (T + dT - T)}{dr}$$

$$\frac{dr}{r} = -\frac{k 2\pi L \, dT}{H}$$

Now we can integrate from the inner edge of the insulation to the outer. H is constant in r because the heat in to each layer equals the heat flow out.

$$\int_{d/2}^{d/2+d} \frac{dr}{r} = -\frac{k 2\pi L}{H} \int_{T_{H_2O}}^{T_{room}} dT$$

$$\ln\left(\frac{3/2\, d}{1/2\, d}\right) = -\frac{k 2\pi L}{H} \left(T_{room} - T_{H_2O}\right)$$

$$H = -\frac{k 2\pi L}{\ln(3)} \left(T_{room} - T_{H_2O}\right)$$

$$H = \frac{-(0.05 \ W m^{-1} K^{-1})(2\pi)(5 \ m)(-80 \ K)}{\ln(3)}$$

$$H = +114 \ W$$

(b) [2 points] What is the rate of change of entropy of the hot water? Treat the hot water as a reservoir of which the temperature is changed very little.

For a reversible process, $S = \int \frac{dQ}{T}$. Since S is a state function, we can create a reversible process that draws 114 W of heat from the water per second, and find S for this process.

$$\frac{dS_{H_2O}}{dt} = \frac{d}{dt} \int \frac{dQ}{T_{H_2O}} = \frac{1}{T_{H_2O}} \frac{d}{dt} \int dQ \quad \text{since } T_{H_2O} \text{ is constant}$$

$$= \frac{1}{T_{H_2O}} \frac{dQ}{dt} = \frac{-H}{T_{H_2O}}$$

$$\frac{dS_{H_2O}}{dt} = \frac{-114 \ W}{(273 + 90) \ K} = -0.314 \ W K^{-1}$$

(c) [2 points] What is the rate of change of entropy of the insulation?

All the heat that flows in to the insulation flows out. Again, we can construct a reversible process and use $S_{ins} = \int \frac{dQ}{T}$. But $dQ = 0$: no heat is gained or lost by any of the layers of insulation. Hence $S_{ins} = 0 \Rightarrow \frac{dS_{ins}}{dt} = 0$.

(d) [2 points] What is the rate of change of entropy of the room?

The heat that flows out of the water flows into the room. The temperature of the room doesn't change, so

$$\frac{dS_{room}}{dt} = \frac{d}{dt}\int \frac{dQ}{T_{room}} = \frac{1}{T_{room}}\frac{d}{dt}\int dQ = \frac{1}{T_{room}}\frac{dQ}{dt} = \frac{+H}{T_{room}}$$

$$\frac{dS_{room}}{dt} = \frac{114W}{(273+10)\,K} = 0.403 \ W\,K^{-1}$$

(e) [2 points] What is the rate of change of entropy of the universe? Is the second law of thermodynamics valid for this process?

$$\Delta S_{universe} = \Delta S_{H_2O} + \cancel{\Delta S_{ins}}^{0} + \Delta S_{room}$$

$$\frac{dS_{univ}}{dt} = \frac{dS_{H_2O}}{dt} + \cancel{\frac{dS_{ins}}{dt}} + \frac{dS_{room}}{dt}$$

$$\frac{dS_{univ}}{dt} = -0.314 + 0 + 0.403 \ W\,K^{-1}$$

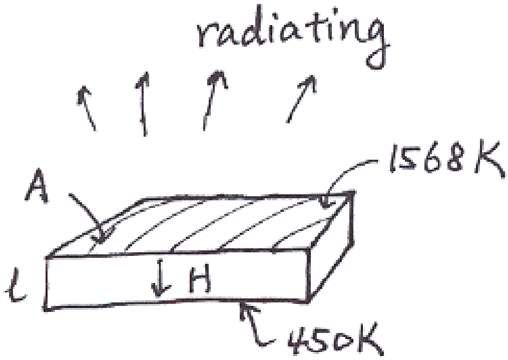
$$\frac{dS_{univ}}{dt} = 0.0890 \ W\,K^{-1} > 0$$

So the rate of change of entropy of the universe is greater than zero, as the second law tells us it will be for any real process.

4. During re-entry the space shuttle is slowed down by coming in at a large angle of attack, that is the angle between the wing and the direction of air flow. However, friction between the surface and the thin air in the upper atmosphere also generates a large amount of heat.

At the beginning of the re-entry the space shuttle travels at a speed of Mach 25, that is, 5.4 km/s at an altitude of 61 km. At this height, the temperature is 116 K, and the pressure is 10^{-4} atm.

(a) [10 points] The highest temperature of the silica tiles on the shuttle that is allowed to reach is 1568 K, whereas the aluminum frame shielded by the tiles is allowed to go up to 450 K. What is the amount of heat that can be dissipated from the surface of a 15 cm by 15 cm tile when all the materials reach their maximum allowed temperature? The thermal conductivity of silica at 1568 K and 10^{-4} atm is 0.073 BTU·ft^{-1}·hr^{-1}·°F^{-1}. The thickness of the tile is 12.5 cm. The emissivity of the surface of the tile is 0.85.



Heat dissipates from the top surface through both radiation and heat conduction.

$$\frac{dQ}{dt} = e A \sigma T^4 + \frac{k A \Delta T}{l}$$

$$= (0.85)(0.15\,m)^2 \cdot (5.67\times10^{-8} W\cdot m^{-2} K^{-4})(1568\,K)^4$$

$$+ \left[\frac{0.073 \cdot 1055 J}{(0.3048\,m)\cdot(3600\,s)\cdot(\frac{5}{9}K)}\right] \frac{(0.15\,m)^2 (1568-450)K}{(0.125\,m)}$$

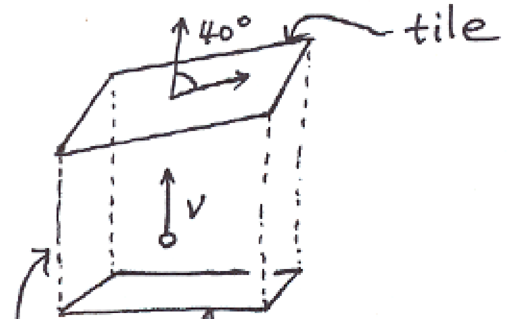
$k = 0.126337 \frac{W}{m\cdot K}$

$$= 6554.94\ J/s + 25.4242\ J/s$$

$$= \boxed{6580\ J/s}$$

(b) [5 points] What is the actual power dumped to the 15 cm by 15 cm tile at the beginning of the re-entry if the angle of attack is 40°? The density of air at the altitude of 61 km is about 3×10^{-4} kg·m^{-3}.

In the rest frame of the shuttle wing, air molecules strike the tile at the shuttle velocity $v = 5.4$ km/s at an angle of 40°. Assume all the kinetic energy of the incoming gas molecules is converting into heat.



area normal to gas flow

$A' = (0.15\text{m})^2 \sin 40°$

mass density $\rho = m \cdot n$

where n is number density.

- Each particle has kinetic energy $\frac{1}{2} m v^2$
- The number of particles striking the wing per second is $n \cdot A' \cdot v$
- Hence, the actual power dumped into the tile is

$$\frac{dQ}{dt} = \left(\tfrac{1}{2} m v^2\right)\left(n \cdot A' \cdot v\right)$$

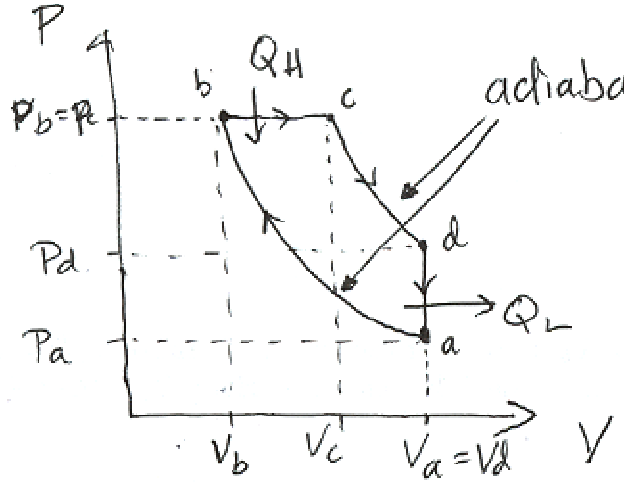
$$= \tfrac{1}{2} \cdot \underset{\downarrow}{(m \cdot n)} \cdot \left((0.15\text{m})^2 \sin 40°\right) \cdot v^3$$

$$= \tfrac{1}{2} \left(3 \times 10^{-4} \text{kg}\cdot\text{m}^{-3}\right)(0.15\text{m})^2 (\sin 40°) \cdot \left(5.4 \times 10^3 \tfrac{\text{m}}{\text{s}}\right)^3$$

$$= 341604 \text{ J/s}$$

5. A diesel engine employs four processes to complete a cycle. We shall model such an engine as a quasi-static reversible engine using an ideal gas. From an initial state a the gas is compressed adiabatically to state b. Then the gas is expanded at constant pressure to state c. During this transition an amount of heat Q_H is taken in. The process that goes from state c to state d is adiabatic. In the end of the cycle, heat of amount Q_L is ejected as exhaust during the isochoric process that goes from state d to state a.

(a) [5 points] Draw the cycle on a PV diagram with all the relevant quantities labelled clearly.



adiabadic $\begin{cases} a \to b \\ c \to d \end{cases}$

(b) [15 points] Determine the efficiency of this ideal engine in terms of the volumes of the states and the γ factor.

$$e = 1 - \frac{Q_L}{Q_H} \qquad Q_L = C_V \Delta T_{da} \qquad Q_H = C_P \Delta T_{bc}$$

$$Q_L = C_V\left(\frac{P_d V_d}{Nk} - \frac{P_a V_a}{Nk}\right) = \frac{C_V}{Nk} V_a (P_d - P_a) \quad [V_a = V_d]$$

$$Q_H = C_P\left(\frac{P_c V_c}{Nk} - \frac{P_b V_b}{Nk}\right) = \frac{C_P}{Nk} P_c (V_c - V_b) \quad [P_c = P_b]$$

$$e = 1 - \frac{\frac{C_V}{Nk} V_a (P_d - P_a)}{\frac{C_P}{Nk} P_c (V_c - V_b)} = 1 - \frac{V_a (P_d - P_a)}{\gamma P_c (V_c - V_b)} \quad \left[\gamma = \frac{C_P}{C_V}\right]$$

$c \to d$ and $a \to b$ are adiabadic

$$P_c V_c^{\gamma} = P_d V_a^{\gamma} \qquad P_a V_a^{\gamma} = P_b V_b^{\gamma}$$

$$P_d = P_c \frac{V_c^{\gamma}}{V_d^{\gamma}} = P_b \frac{V_c^{\gamma}}{V_a^{\gamma}} \qquad P_a = P_b \frac{V_b^{\gamma}}{V_a^{\gamma}}$$

$$e = 1 - \frac{V_a \left(P_b \frac{V_c^\gamma}{V_a^\gamma} - P_b \frac{V_b^\gamma}{V_a^\gamma} \right)}{\gamma P_b (V_c - V_b)}$$

$$\boxed{e = 1 - \frac{\left(\frac{V_a}{V_c}\right)^{-\gamma} - \left(\frac{V_a}{V_b}\right)^{-\gamma}}{\gamma \left[\left(\frac{V_a}{V_c}\right)^{-1} - \left(\frac{V_a}{V_b}\right)^{-1} \right]}}$$

(c) [5 points] If the compression ratio, V_a/V_b, is 10, and the expansion ratio, V_a/V_c, is 5, calculate the ideal efficiency of the engine for an ideal diatomic gas. Here, V_i is the volume of the gas at state i.

For an ideal engine $e = 1 - \frac{T_L}{T_H}$. The low and high temperatures occur at point (a) and point (c) respectively.

$$T_H = \frac{P_c V_c}{Nk} \qquad T_L = \frac{P_a V_a}{Nk}$$

$$P_a V_a^\gamma = P_b V_b^\gamma \text{ [still ok ...]} \Rightarrow P_a = P_b \frac{V_b^\gamma}{V_a^\gamma}$$

$$\frac{T_L}{T_H} = \frac{P_a V_a}{P_c V_c} = \frac{P_b (V_b/V_a)^\gamma V_a}{P_c V_c} = (V_b/V_a)^\gamma \frac{V_a}{V_c} \qquad [P_b = P_c]$$

$$\boxed{e = 1 - \frac{(V_a/V_c)}{(V_a/V_b)^\gamma}} \qquad \gamma = \frac{d+2}{d} = \frac{5+2}{5} = \frac{7}{5}$$

End of Examination

$$\boxed{e = 1 - \frac{5}{10^{7/5}} = .800946}$$