University of California, Berkeley
Physics 137A, Lecture 001, Fall 2011 (*Xiaosheng Huang*)

Midterm Examination
Friday, 10/14/2011

Total Points: 100

Note: *You are allowed one handwritten formula card (3½" by 5", double sided). No calculators or any other electronic devices are permitted. Show all work, and take particular care to explain what you are doing. Please use the symbols described in the problems and appropriate constants, define any new symbols that you introduce, and label any drawings that you make.*

1. (30 pts.) A particle in a harmonic oscillator potential starts out in the state

$$\Psi(x,0) = A[2\psi_0(x) + 3\psi_1(x)]$$

with $\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\xi^2/2}$ and $\psi_1(x) = \sqrt{2}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \xi e^{-\xi^2/2}$, where $\xi = \sqrt{\frac{m\omega}{\hbar}}x$

a) Find A. You may assume it is real.
b) Construct $\Psi(x, t)$ and $|\Psi(x, t)|^2$.
c) Find $\langle x \rangle$.
d) Find $\langle p \rangle$ directly, without using $m\frac{d\langle x \rangle}{dt}$.
e) If you measured the energy of this particle, what values might you get, and with what probabilities?

You may find this integral helpful:

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

2. (30 pts.) Consider a particle of mass m that is prepared with the wavefunction $\Psi(x, 0) = -A(x^2 - a^2)$, in an infinite square well of width $2a$ centered on $x = 0$. The constant A is real and positive. Suppose a billion of these systems are prepared identically and the energy of the particle in each system is measured at $t = 0$.

a) Determine A.
b) Write down the normalized eigenfunction for the ground state for the well. Find the percentage of the particles you would expect to be found in this state.
c) Find the average value of all the energy measurements at $t = 0$. (*Hint*: There are more than one ways to calculate this quantity. What's the easiest way?)
d) What will the average value of all the energy measurements be for $t > 0$?

e) Suppose the energy of the particle in one particular system is measured to be E_3 at $t = 0$ (where E_3 is the energy of the second excited state). What is the wavefunction of this particle one hour later?

3. (20 pts.) Imagine a bead of mass m that slides frictionlessly around a circular wire ring of circumference L. This is just like a free particle except that $\psi(x+L) = \psi(x)$. Find the stationary states, with appropriate normalization, and the corresponding allowed energies. In imposing the normalization condition you only need to integrate from 0 to L. Note that there are two independent solutions for each energy E_n corresponding to clockwise and counter-clockwise circulation; call them $\psi_n^+(x)$ and $\psi_n^-(x)$.

4. (20 pts.) This problem only requires qualitative answers. Throughout this problem, (i) the mass of the particle is m; (ii) $E > V_2 > V_1$; and (iii) $V = 0$ for $x < 0$.

Suppose the transmission coefficient for a single potential step V_1 is T_1 (Fig. 1 (a)), and that for a single potential step V_2 is T_2 (Fig. 1 (b)),
a) Offer a physical explanation as to why you would expect $T_1 \geq T_2$.

The transmission coefficient for the double step potential shown in Fig. 1 (c) is T. The width of the potential V_1 is a.
b) Offer a physical explanation as to why $T \geq T_2$. T_2 has the same definition as above. (*Hint*: How do you expect T varies as a function of a?)
[This is why on modern camera lenses there is an optical coating — to enhance the transmission of light.]

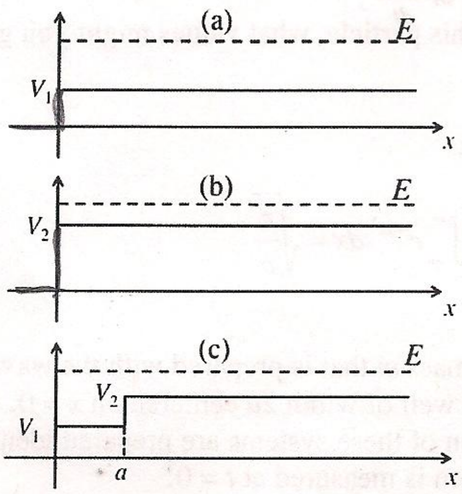

Fig. 1

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