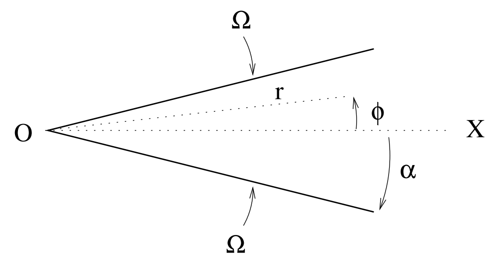
UNIVERSITY OF CALIFORNIA, BERKELEY
MECHANICAL ENGINEERING
ME106 Fluid Mechanics **NAME** ______________________________
2nd Test, S12 *Prof S. Morris* Overall Mean: 132.5, STD: 37.6

1. (65) Abruptly closing an open book makes a bang as air is expelled rapidly from the gap between the pages. In the figure, each cover rotates around the axis O with angular velocity $\pm\Omega$; the instantaneous angle between the covers is 2α, and (r, ϕ) are plane polar coordinates.



Problem1
Mean: 43.2
STD: 16.5

(a) Assuming that $v_\phi = Ar\sin 2\phi$, use the boundary conditions and the continuity equation for incompressible flow

$$\frac{1}{r}\frac{\partial(rv_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\phi}{\partial \phi} = 0$$

to find v_r in terms of r, ϕ, Ω and α. (A must not appear in your answer.) Note that v_r must be finite at $r = 0$.

(b) To find A, you will have imposed a boundary condition at $\phi = \alpha$; what is the name of this boundary condition?

SOLUTION

(a) Substituting for v_ϕ into the continuity equation, we obtain

$$\frac{1}{r}\frac{\partial(rv_r)}{\partial r} + 2A\cos 2\phi = 0.$$ +15

Multiplying by r and integrating, we obtain

$$rv_r = -Ar^2\cos 2\phi + G(\phi);$$ +15

$G(\phi)$ is an arbitrary function of integration.

As stated, v_r must be finite at $r = 0$; consequently, $G(\phi) = 0$.

+15

To determine A, we note that at $\phi = \alpha$, $v_\phi = -\Omega r$; consequently,

+15 Equate Vφ=Ωr
Idea: 10, Execution: 5
If set G(φ)=0 without explanation 5/15

$$-\Omega r = Ar\sin 2\alpha \quad\Rightarrow\quad A = -\frac{\Omega}{\sin 2\alpha}$$

Hence

$$v_r = \Omega r\frac{\cos 2\phi}{\sin 2\alpha}$$

(b) Kinematic, or no penetration, boundary condition. +5 if mentioned also "no slip" 4/5

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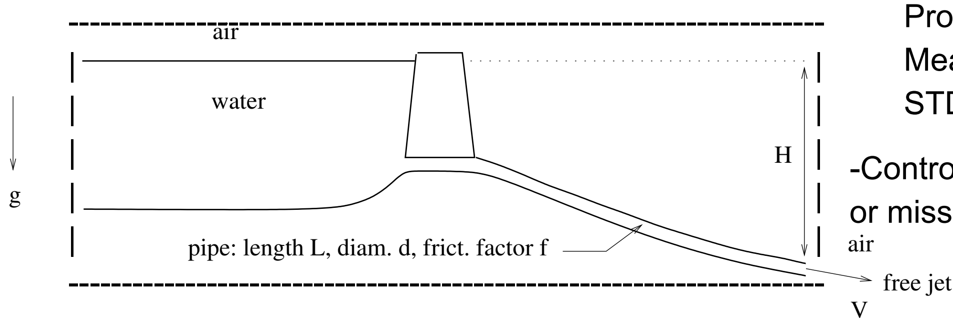
2. (65) (a) Water drains from the reservoir to atmospheric pressure through a pipe of length L and diameter d. Assuming quasi–steady flow, and that the power loss in the pipe is given by $\frac{1}{2}\dot{m}fV^2L/d$, derive the expression giving the velocity V in the pipe as a function of d, L, g, elevation change H, and friction factor f. For credit, you must include a sketch of your control volume, and you must explain briefly your assumptions.

(b) Hence find V if $L = 300$ km, $H = 1$ km, $d = 1$m and $f = 0.03$. (You may assume that $g = 10$ m^2/s; for credit, your answer must be given as a decimal number and be accurate to within around 10%.)



Problem2
Mean: 59.0
STD: 9.9

-Control Surface is incorrect or mission: -5

SOLUTION

(60) (a) Balance of mechanical energy on the control volume shown:

$$\dot{m}[\tfrac{1}{2}V^2 + (p/\rho) + gz]_1^2 = \text{S.P.} - \text{power loss.}$$

(-10) if not correct

Definition of the friction factor:

$$\text{Power loss} = \tfrac{1}{2}\dot{m}fV^2\frac{L}{d}$$

Simplifying the energy balance: $V_1 \ll V_2 = V$; $p_1 = p_2$ (atmospheric); $\dot{m}$ cancels from the energy balance.

if not, (-5) if not, (-5)

$$\tfrac{1}{2}V^2 - gH = -\tfrac{1}{2}fV^2\frac{L}{d}$$

Solving for V^2, we obtain

$$\tfrac{1}{2}\left(1 + f\frac{L}{d}\right) = gH.$$

That is

$$V = \sqrt{\frac{2gH}{1 + f(L/d)}}$$

-minus sign under the square root without any notice, (-10)
-Dimensional error with correct approach (40/60)

(5) (b) With the numbers given

$$V = \sqrt{\frac{2 \times 10 \times 10^3}{1 + (0.03 \times 3 \times 10^5)}}, = \sqrt{\frac{2 \times 10^4}{0.9 \times 10^4}}, \simeq \sqrt{2} \simeq 1.4\text{m/s}$$

(-2) if it is not correct within around10%

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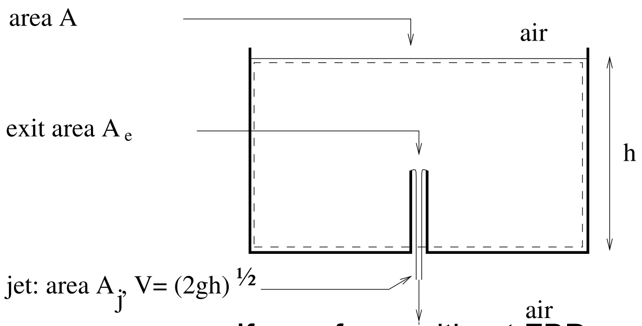
3. (70) Water drains from a large tank through a small exit of area $A_e \ll A$ located within the tank interior far from all walls; as a result, p is hydrostatic on the walls. Assuming that the jet velocity $V = \sqrt{2gh}$, and using a balance momentum on the control volume shown, find the ratio A_j/A_e of the jet area to the exit area.



Problem3
Mean: 30.3
STD: 22.9

If zero force without FBD, you can't have any partial credit.

SOLUTION

According to the balance of vertical momentum, the resultant vertical force (downwards) is equal to the net flux of vertical momentum out of the control volume. Proper Mom. Bal.: (+10)

Because the tank is given to be large, the momentum flux through the upper surface of the control volume can be taken as negligibly small; the flux out through the jet is given by $\rho V^2 A_j$. Mom. Flux.: (+10)

The resultant vertical force acting the water consists of two parts: its weight $\rho g A h$ (downward), and the resultant (upward) hydrostatic pressure force $\rho g(A - A_e)$ acting on the horizontal faces of the control surface. The resultant of these two is equal to $\rho g A_e$. Weight: (+20), Pressure: (+20)

Equating the momentum flux to the resultant force, we obtain $\rho V^2 A_j = \rho g A_e$. Cancelling the common factor of ρ, then substituting for V^2, we obtain

$$\frac{A_j}{A_e} = \tfrac{1}{2}.$$ Result (+10)

The momentum balance requires the jet area to be half the exit area:

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