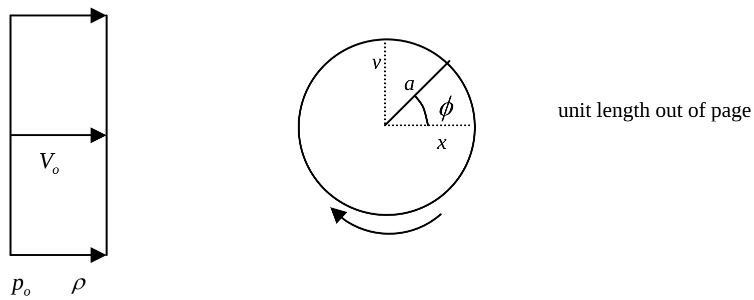
UNIVERSITY OF CALIFORNIA, BERKELEY
MECHANICAL ENGINEERING
ME106 Fluid Mechnics
1st Test, S12 *Prof S. Morris*

1. (65) Far from the spinning *cylinder*, the air of density ρ has uniform velocity $V_o\mathbf{i}$ and pressure p_o. On the cylinder, the pressure is given as a function of angle ϕ by $p(\phi) - p_o = -4\rho V_0^2 (J + \sin\phi)^2$; J is a given constant. The aim is to find the component of the resultant pressure force acting *parallel* to the free stream $V_o\mathbf{i}$.



(a) Derive the expression giving F_x as an integral of $p(\phi)$ with respect to ϕ.

(b) Evaluate your integral to determine F_x.

(c) On a single sketch, show $J + \sin\phi$ and $(J + \sin\phi)^2$ as functions of ϕ; then interpret your answer to part (b) using that sketch. For full credit, all curves and axes on your sketch must be clearly labeled.

$$\textbf{Given: } n(\cos\phi)(\sin^{n-1}\phi) = \frac{d}{d\phi}(\sin^n\phi)$$

2. (65) (a) Write the formula for the material derivative $\frac{df}{dt}$ of an arbitrary function $f(x, y, z, t)$.

(b) Using the formula from part (a), evaluate $\frac{dx}{dt}$; to receive credit, you must explain briefly the values you give to each term in the expression for $\frac{dx}{dt}$.

(c) For the flow given by $\mathbf{V} = (Kx + Ly)\mathbf{i} + (Lx - Ky)\mathbf{j}$, find the fluid acceleration **a**. (Hint: **a** || **r**.)

3. (70) At point 1 on the surface of the airfoil, the pressure p is given by $(p - p_\infty)/(\frac{1}{2}\rho V_\infty^2) = -3$. Find the ratio of the flow speed at that point to V_∞. To receive credit, you must explain your logic; a formula and a number is not enough.