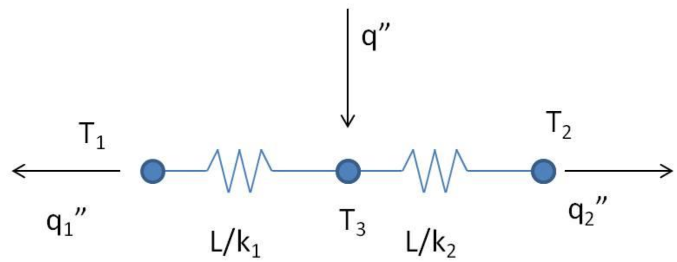
ME 109 Midterm 1 Solutions

1.)

The two-material wall can be depicted as two resistors in series, while the thin film heater adds a heat flux at the border where the two materials meet:



From an energy balance: $q_1'' + q_2'' = q''$

$q_1'' = \frac{T_3 - T_1}{L/k_1}$, $q_2'' = \frac{T_3 - T_2}{L/k_2}$

$q'' = \frac{T_3 - T_1}{L/k_1} + \frac{T_3 - T_2}{L/k_2}$

$q''L + T_1 k_1 + T_2 k_2 = T_3 (k_1 + k_2)$

$T_3 = (q''L + T_1 k_1 + T_2 k_2)/(k_1 + k_2)$

$= [10^6 \text{ W/m}^2 (0.01 \text{ m}) + (100°\text{C})(20\text{W/mK}) + 20°\text{C}(5\text{W/mK})]/(25 \text{ W/mK}) = 484 °\text{C}$

2.)

First check to see if lumped capacitance can be used to solve the problem:

Bi = $(hL_c)/k = (hr_o/3)/k$ = [(100W/m^{2}K)(1.5 cm/3)] / 1 W/mK = 0.5

0.5 < 0.1 → Lumped capacitance cannot be used

Check to see if the approximate analytical solutions can be used:

$F_o = \alpha t/r_o^2$ = 1.33 > 0.2 → One term approximations are okay to use

For a sphere:

$$\theta_o^* = \frac{T(r=0) - T_\infty}{T_i - T_\infty} = C_1 \exp\left(-\zeta_1^2 F_o\right)$$

$$\theta^* = \frac{T(r^*) - T_\infty}{T_i - T_\infty} = \theta_o^* \frac{1}{\zeta_1 r^*} \sin\left(\zeta_1 r^*\right)$$

Where $r^* = r/r_o$ = r/(1.5 cm)

To get C_1 and ζ_1, calculate Bi and look up values in Table 5.1

Use Bi = $(hr_o)/k$ = 1.5

From Table 5.1:

For Bi = 1.0 → ζ_1 = 1.5708, C_1 = 1.2732

For Bi = 2.0 → ζ_1 = 2.0288, C_1 = 1.4793

Interpolate to get:

For Bi = 1.5 → ζ_1 = 1.7998, C_1 = 1.3763

$\theta_o^* = \frac{T(r=0) - T_\infty}{T_i - T_\infty} = C_1 \exp\left(-\zeta_1^2 F_o\right)$ = 0.0183 → T(r=0) = 294.87°C

$\theta^*\left(r^* = \frac{1\ cm}{1.5\ cm}\right) = \frac{T(r^*) - T_\infty}{T_i - T_\infty} = \theta_o^* \frac{1}{\zeta_1 r^*} \sin(\zeta_1 r^*) = 0.0142$ → T(r = 1 cm) = 296.015°C

Energy out of the sphere is Q:

$\frac{Q}{Q_o} = 1 - \frac{3\theta_o^*}{\zeta_1^3}\left[\sin(\zeta_1) - \zeta_1 \cos(\zeta_1)\right]$ = 0.9870

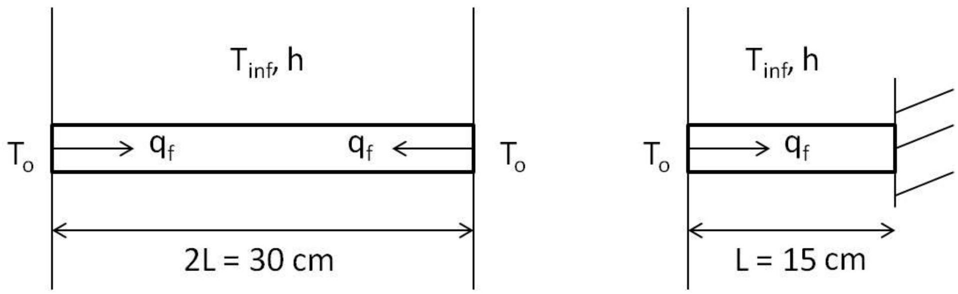
$Q_o = \rho c V(T_i - T_\infty) = \left(\frac{k}{\alpha}\right) V (T_i - T_\infty)$ (from α=k/(ρc))

= -3.958 x 10^3 J

Q = -3.906 x 10^3 J → Energy into the sphere is 3.906 x 10^3 J

3.)

The problem can be simplified by just looking at half of the fin and using symmetry to make the end insulated as shown below:



For an adiabatic tip condition, q_f can be determined (given in Table 3.4):

$$q_f = M \tanh(mL), \quad M = \sqrt{hPkA_c}\,\theta_b, \quad m = \sqrt{\frac{hP}{kA_c}}$$

using $P = 2\pi r$, $A_c = \pi r^2$, and $\theta_b = T_o - T_{inf}$:

$$q_f = 11.75 \text{ W}$$

$$2q_f = \text{total heat to fin} = 23.5 \text{ W}$$