CE100
Midterm Examination
Spring 2012
Friday, February 24, 2012

Name Solutions

Student I.D. ____________________

This exam is open book and open notes. You will be given fifty (50) minutes to complete two problems. Space is provided on each page for your solution, the back of the pages may also be used. Note that the first problem is worth 40 points and the 2nd is worth 10; allocate your time accordingly.

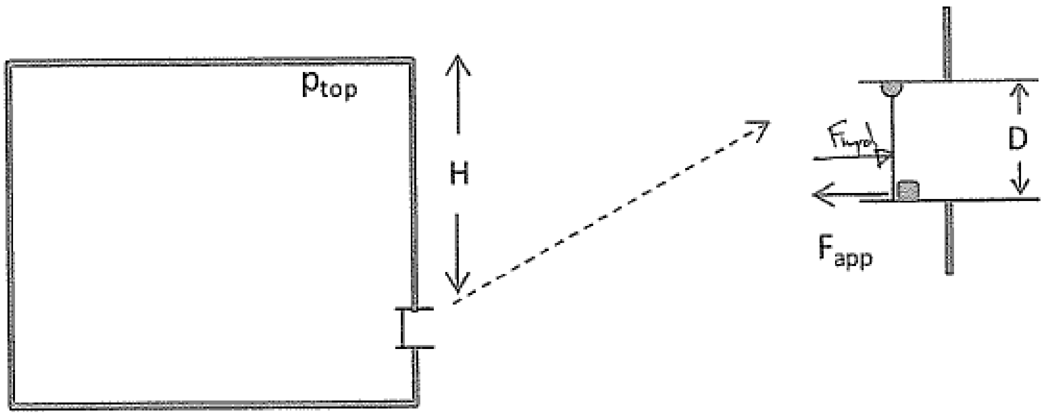
State clearly any assumptions you use in the solutions. Good Luck!

For reference:

Atmospheric Pressure = p_{atm} = 100 kPa
Gravitational Acceleration = g = 9.8 m/s^2

Problem 1 (40 points):

The following closed large tank contains water and has a gate that can be used to release flow. The gate (shown in detail on the right) is designed to be held closed by hydrostatic forces, and is opened through the application of a force applied at its bottom edge (shown in the detailed diagram on the right). The entire outside of the tank is at atmospheric pressure.



The pressure at the top of the tank, ptop = 49 kPa gage and the height of the tank from the top of the tank to the top of the gate is H = 5 meters. The diameter of the outflowing pipe, which is cylindrical, is D = 0.5 meters.

A: Determine the magnitude of the force required to open the gate under these conditions.

B: After the gate is open, determine the volumetric flow rate, Q, out of the chamber through the open gate. You may assume that the flow emerges from the tank as a free jet.

A

Magnitude of hydrostatic force on gate = $p_c \cdot A$

$P_c = P_{top} + (H + \frac{D}{2})\gamma$; $A = \frac{\pi}{4} D^2$

$F_{hyd} = [P_{top} + \gamma(H + \frac{D}{2})]\frac{\pi}{4} D^2$

Note: $y_c = \frac{P_{top}}{\gamma} + H + \frac{D}{2}$

Line of Action: $y_R = y_c + \frac{I}{y_c A} \Rightarrow y_R - y_c = \frac{I}{y_c A} = \frac{\frac{\pi}{4}(\frac{D}{2})^4}{y_c A} = \frac{\frac{\pi}{16} D^2}{[\frac{P_{top}}{\gamma} + H + \frac{D}{2}]}$

$l_{hyd} = \frac{D}{2} + (y_R - y_c) = \frac{D}{2} + \frac{\frac{\pi}{16} D^2}{[\frac{P_{top}}{\gamma} + H + \frac{D}{2}]}$

Moment Balance: $D \cdot F_{app} = l_{hyd} \cdot F_{hyd} \Rightarrow F_{app} = \left[\frac{1}{2} + \frac{\frac{\pi}{16} D}{[\frac{P_{top}}{\gamma} + H + \frac{D}{2}]}\right] \cdot [P_{top} + \gamma(H + \frac{D}{2})]\frac{\pi}{4} D$

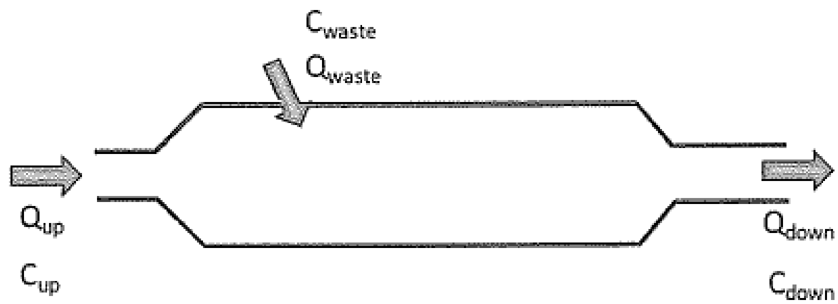
B

Bernoulli: $\frac{P_{top}}{\gamma} + z_{top} + \frac{V_{top}^2}{2g} = \frac{P_{out}}{\gamma} + z_{out} + \frac{V_{out}^2}{2g}$ ($z_{top} \to H$, $V_{top} \to 0$, $P_{out} \to 0$, $z_{out} \to 0$) $\Rightarrow V_{out} = \sqrt{2gH + 2\frac{P_{top}}{\rho}}$

$Q_{out} = V_{out} \cdot A_{out} = \frac{\pi}{4} D^2 \sqrt{2gH + 2\frac{P_{top}}{\rho}}$

Problem 2 (10 points):

The Stockton Deep Water Shipping Channel (SDWSC) is a deep, slow-moving part of the San Joaquin River that has persistent problems with algal blooms and oxygen depletion. You can picture this channel as a wide, rectangular channel with one primary inflow and one outflow, but also a secondary inflow from a wastewater return in Stockton, as shown in the following sketch:



The flow from upstream, Q_{up}, has concentration of algae, C_{up}. Downstream, the volumetric flow rate is Q_{down} and the algae concentration is C_{down}. The wasterwater inputs have volumetric flow rate Q_{waste} and algae concentration C_{waste}. Analyses of the mass of algae in the SDWSC must account for its growth within the SDWSC as well as fluxes into and out of the SDWSC. Set up the Reynolds transport theorem for this case and describe what each term represents. (You will not be solving the equation, just setting it up and describing it.)

Mass of Algae following system due to growth = "Growth"

$$\frac{dM_{sys}}{dt} = \text{"Growth"} = \frac{dM_{cv}}{dt} + \sum_{i=1}^{3} \rho C_i (\pm Q_i)$$

$$\Rightarrow \frac{dM_{cv}}{dt} = \text{"Growth"} + \underbrace{\overbrace{\rho C_{waste} Q_{waste} + \rho C_{up} Q_{up}}^{\text{inflows}} - \overbrace{\rho Q_{down} C_{down}}^{\text{outflows}}}_{\text{advective fluxes of algae}}$$

- $\frac{dM_{cv}}{dt}$: change in algal mass in CV
- "Growth": growth rate following volume of fluid
- $\rho C_{waste} Q_{waste} + \rho C_{up} Q_{up} - \rho Q_{down} C_{down}$: advective fluxes of algae