Midterm 2 Ma 53 Fall '09 Prof

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1. z is a function of x and y defined implicitly by $\sin x \sin y \sin z = \frac{1}{4}$.

Compute $z_x\left(\frac{\pi}{4}, \frac{\pi}{6}\right)$ and $z_y\left(\frac{\pi}{4}, \frac{\pi}{6}\right)$

assuming $z\left(\frac{\pi}{4}, \frac{\pi}{6}\right) = \frac{\pi}{4}$.

2. Given $F(x, y)$ define $f(r, \theta)$ by

$f(r, \theta) := F(r\cos\theta, r\sin\theta)$.

Suppose $f_r\left(\sqrt{2}, \frac{\pi}{4}\right) = 1$, $f_\theta\left(\sqrt{2}, \frac{\pi}{4}\right) = -1$.

What are $F_x(1, 1)$ and $F_y(1, 1)$?

3. a) Compute $\frac{\nabla f}{f}$ for $f(x, y, z) = \sin x \sin y \sin z$.

b) Find the equation of the tangent plane to the surface $\sin x \sin y \sin z = \frac{1}{4}$ at $(x, y, z) = \left(\frac{\pi}{4}, \frac{\pi}{6}, \frac{\pi}{4}\right)$.

c) Use differentials to estimate

$$r := \frac{f\left(\frac{\pi}{4}+.01,\ \frac{\pi}{6},\ \frac{\pi}{4}+.01\right) - f\left(\frac{\pi}{4}, \frac{\pi}{6}, \frac{\pi}{4}\right)}{f\left(\frac{\pi}{4}, \frac{\pi}{6}, \frac{\pi}{4}\right)}.$$

4. a) Compute $\iint_D e^{-x}e^{-y}\,dA$ where D is the wedge in first quadrant with $mx < y < m'x$. Here, $m > 0$ and $m' > m$ are given constants.

b) If the distribution of $x > 0$ is e^{-x} and the distribution of $y > 0$ is e^{-y}, what fraction of (x, y) in first quadrant have $\theta := \arctan\frac{y}{x}$ between $\frac{\pi}{6}$ and $\frac{\pi}{4}$?

5. Evaluate $\iiint_E e^{-x^2-y^2-z^2} \frac{x^2}{x^2+y^2}\,dV$, where E is the (infinite) cylinder $x^2 + y^2 < 1$.