Physics 7C Midterm 2

October 26, 2010

Please record your answers in a blue book. It is your responsibility to make your answers clear. Write your name and discussion SECTION NUMBER on the front. If you do not know your section number, please write down the name of your GSI.

Problem 1 (20 pts) A coherent plane electro-magnetic wave of wavelength λ is incident on 3 equally spaced slits. Assume the width of each slit is very small in comparison to slit separation d. The light passing through these slits hits a screen very far away. Focus on a point P on the screen at an angle θ with respect to the normal to the slits and answer the following in terms of λ , d and θ, and E_0, the electro-magnetic field amplitude on the screen, produced by a single such slit.

a. (2 pts) Write down the phase difference δ, between the electro-magnetic waves produced by a pair of neighboring slits as a function of d, λ and θ.

b. (3 pts) Draw the phasor diagram for arbitrary δ. Clearly indicate δ on your diagram. What does the length of each phasor correspond to in terms of the quantities given? Adding up all the phasors, what is the meaning of the resulting vector?

c. (6 pts) Draw the phasor diagrams corresponding to

I. principal (i.e. global) maxima

II. minima

III. secondary (i.e. local) maxima

What values of $\delta \in [-\pi, \pi)$ do these correspond to? (Note that there can be more than one value of δ corresponding to some of the above cases).

d. (3 pts) What are the values of $\frac{E}{E_0}$ for the above situations (i.e. cases (I),(II), and (III)) where E is the net amplitude of the electro-magnetic field on the screen for the corresponding δ and E_0 is defined to be the field amplitude due to a single slit.

e. (3 pts) Sketch a graph of the relative intensity $\frac{I}{I_0}$ on the screen as a function of $\delta \in [-\pi, \pi)$ given your results from above. Here, I is the net intensity and I_0 is the intensity due to a single slit. How is $\frac{I}{I_0}$ related to $\frac{E}{E_0}$?

f. (3 pts) How would the graph change if the slits had a relatively large thickness of $\frac{d}{4}$?

Problem 2 (10 pts) You are given three Polaroid sheets, with their transmission axes labeled by the manufacturer (the transmission axis here is the axis along which the sheet polarizes incident light.) Take a source of unpolarized light of intensity I.

a. (3 pts) Start with just one sheet with its axis in the $\hat{x}$ direction. As light passes through the sheet, what is the resulting intensity?

b. (2 pts) Add another sheet on top of the sheet in a) (i.e. closer to the source). Arrange its axis in the $\hat{y}$ direction, and shine light through the sheet. What is the intensity now?

c. (5 pts) Add the third sheet, in between the sheets in a) and b). Choose the direction of the axis to make an angle α with the $\hat{x}$ direction. What will be the intensity of the resulting light? What will be its direction of polarization? You must explicitly show your derivation of the intensity formula as a function of α to receive full credit.

Problem 3 (25 pts) Alice embarks on a space voyage. She travels at speed $\frac{9}{10}c$ from Earth to a Star 10 light-years away (i.e. it takes light 10 years to get there) as measured in the common rest frame of the Earth and the star. Alice spends a year on the star studying the local civilization (as measured in Earth-Star frame), and then she embarks on the voyage back. Her hosts give her a faster space ship, so on her return trip, she travels at speed $\frac{99}{100}c$. Let A be the event of Alice's departure from Earth, B her arrival to the star, C her departure from the star, and D her arrival back to Earth.

a. (3 pts) Draw the space-time diagram from the rest frame of the Earth-Star system, indicating Alice's world-line, and events A, B, C, D.

b. (4 pts) Find the distance and the time interval between events A and B in the Earth-Star rest frame. Find the distance and the time interval between events C and D in the Earth-Star rest frame. How long did Alice's trip take from the perspective of her mother on Earth? What are the (x, ct) coordinates of events A, B, C, D in this frame?

c. (5 pts) Find the time interval and space interval between events A and B in Alice's rest frame using Lorentz transformations. Check your answer against computing an invariant space-time interval between events A,B.

d. (3 pts) Find the time interval between events (B, C) and (C, D) in Alice's frame as well. How long did the trip take from Alice's perspective?

e. (4 pts) How much older is Alice when she comes back? (If she was 20 years old when she started out, will she come back alive? If so, will her face be wrinkled?). Explain how to reconcile this with your answer in b. Is this in conflict with the principle of relativity – the idea that there is no preferred inertial reference system?

f. (2 pts) On the space-time diagram, draw a slice of simultaneity for Alice on her way out. What values of x, t (the coordinates of the Earth-Star reference system) does that correspond to?

g. (4 pts) What is the Sun-star distance from Alice's perspective? What is the velocity of the Star in Alice's reference frame? Using this information find how long will it take for the Alice to land on the Star. Compare this to your answer in c).