1.

Use the Gram-Schmidt process to find an orthonormal basis for $W = Span\,\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$, where:

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \mathbf{u}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

2.

Suppose A is a 3×3 matrix with real entries. Are the following statements true or false? Justify your answer.

2a.

If A has 3 linearly independent eigenvectors then it is diagonalizable.

2.b

If A is diagonalizable it must have 3 distinct eigenvalues.

2.d

If $A = PDP^{-1}$ then A has the same eigenvalues as D.

2.e

If A is diagonalizable then A is invertible.

3.

Consider $M_{2\times 2}$ the vector space of 2×2 matrices (with standard addition and multiplication by scalars). Let

$$F = \begin{bmatrix} -2 & -1 \\ 4 & 3 \end{bmatrix}$$

Consider the map $T : M_{2\times 2} \to M_{2\times 2}$ such that for each 2×2 matrix A, $T(A) = AF$.

a) Show that T is a linear transformation.

b) Find the matrix associated to T with respect to the basis $\{b_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, b_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, b_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, b_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\}$

4.

Find the equation $y = \beta_0 + \beta_1 x$ of the least squares line that best fits the given data points:

(1,0) , (2,2), (3,1)