Problem 1. (34 points total)

For the direct central impact between $A$ and $B$, (14 points for $A$-$B$ impact)

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$\Rightarrow \quad 2mv + 0 = 2m(v_A)_2 + m\,(v_B)_2 \tag{1}$$

From the coefficient of restitution,

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} = \frac{(v_B)_2 - (v_A)_2}{v} \tag{2}$$

Solve Eqs. (1) and (2) to obtain

$$(v_A)_2 = \frac{v}{3}(2-e) \quad \longrightarrow$$ (3 points for answer)

$$(v_B)_2 = \frac{2v}{3}(1+e) \quad \longrightarrow$$

From geometry,

$$\sin\theta = r/2r \quad \Rightarrow \quad \theta = 30^\circ$$

For the oblique central impact between $B$ and $C$, (20 points for $B$-$C$ impact)

$$m_B(v_B)_{2n} + m_C(v_C)_{2n} = m_B(v_B)_{3n} + m_C(v_C)_{3n}$$

$$\Rightarrow \quad (v_B)_2\cos\theta + 0 = (v_B)_{3n} + (v_C)_{3n} \tag{3}$$

Along the $n$-direction,

$$e = \frac{(v_C)_{3n} - (v_B)_{3n}}{(v_B)_{2n} - (v_C)_{2n}} = \frac{(v_C)_{3n} - (v_B)_{3n}}{(v_B)_2\cos\theta} \tag{4}$$

Solve Eqs. (3) and (4) to get

$$(v_B)_{3n} = \frac{1}{2}(1-e)(v_B)_2\cos\theta$$

$$(v_C)_{3n} = \frac{1}{2}(1+e)(v_B)_2\cos\theta$$

No forces are generated along the $t$-direction, therefore

$$m_B(v_B)_{2t} = m_B(v_B)_{3t} \quad \Rightarrow \quad (v_B)_{3t} = -(v_B)_2\sin\theta$$

$$m_C(v_C)_{2t} = m_C(v_C)_{3t} \quad \Rightarrow \quad (v_C)_{3t} = 0$$

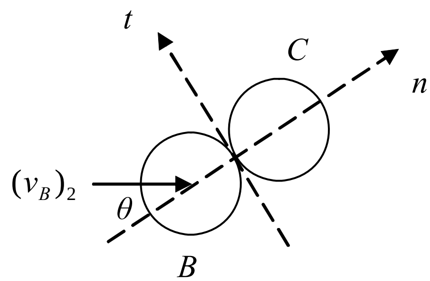
The speed of $A$ after collision is $(v_A)_2$ obtained earlier while the speeds of $B$, $C$ after collision are given by

(3 points for answer)

$$(v_B)_3 = \sqrt{(v_B)_{3n}^2 + (v_B)_{3t}^2} = \frac{v(1+e)\sqrt{4+3(1-e)^2}}{6} \quad \searrow$$

(3 points for answer)

$$(v_C)_3 = (v_C)_{3n} = \frac{v\sqrt{3}(1+e)^2}{6} \quad \nearrow 30^\circ$$




Problem 2. (33 points total) (28 points for angular velocity of *CD*)

Attach an absolute *xy*-frame to *O* with the *x*-axis in a horizontal direction. Since *A* moves in a circle about *O*,

$$\mathbf{v}_A = 3\omega_{AO}\cos 30°\mathbf{i} - 3\omega_{AO}\sin 30°\mathbf{j} = 30\cos 30°\mathbf{i} - 30\sin 30°\mathbf{j}$$

Assume that *CD* has a clockwise angular velocity, which implies that $\boldsymbol{\omega}_{CD} = -\omega_{CD}\mathbf{k}$. The velocity $\mathbf{v}_B$ is perpendicular to *CB* such that

$$\mathbf{v}_B = 4\omega_{CD}\mathbf{i}$$

Since *B* moves in a circle relative to *A*, $\mathbf{v}_{B/A}$ is perpendicular to *AB* and therefore it acts along *CB* when *AB* is horizontal and *CB* is vertical.

$$\mathbf{v}_{B/A} = v_{B/A}\mathbf{j}$$

For points *A*, *B* on link *AB*,

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

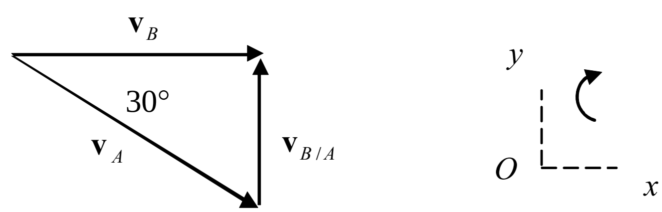
$$\Rightarrow \quad 4\omega_{CD}\mathbf{i} = 30\cos 30°\mathbf{i} - 30\sin 30°\mathbf{j} + v_{B/A}\mathbf{j}$$

$$\Rightarrow \quad 4\omega_{CD} = 30\cos 30°$$

$$\Rightarrow \quad \omega_{CD} = 6.50 \text{ rad/sec}$$ ↻ (3 points for answer)

The velocity $\mathbf{v}_D$ is perpendicular to the arm *CD* such that (5 points for velocity of *D*)

$$\mathbf{v}_D = 6\omega_{CD}\mathbf{i} = 38.97\mathbf{i} \text{ in/sec}$$ → (3 points for answer)




Problem 3. (33 points total)

The rotation of the wheel and the motion of the center *O* are connected by

$$\omega = \frac{v_O}{r}$$ (5 points for angular velocity)

$$\alpha = \frac{(a_O)_t}{r}$$ (5 points for angular acceleration)

Attach a translating *xy*-frame to *O*.

$$\mathbf{a}_O = (\mathbf{a}_O)_t + (\mathbf{a}_O)_n = r\alpha\mathbf{i} + \frac{v_O^2}{R-r}\mathbf{j} = r\alpha\mathbf{i} + \frac{(r\omega)^2}{R-r}\mathbf{j}$$

For $A$ and $O$ on the wheel,

$$\begin{aligned}\mathbf{a}_A &= \mathbf{a}_O + \boldsymbol{\omega}_{AO} \times (\boldsymbol{\omega}_{AO} \times \mathbf{r}_{A/O}) + \boldsymbol{\alpha}_{AO} \times \mathbf{r}_{A/O} \\ &= r\alpha\mathbf{i} + \frac{(r\omega)^2}{R-r}\mathbf{j} + (-\omega\mathbf{k}) \times [(-\omega\mathbf{k}) \times r\mathbf{j}] + (-\alpha\mathbf{k}) \times r\mathbf{j} \\ &= 2r\alpha\mathbf{i} + \frac{(2r-R)r\omega^2}{R-r}\mathbf{j}\end{aligned}$$

(5 points for answer)