Problem 1. (33 points total)

(a) (28 points)

$$v_T = \frac{60}{3.6} = 16.67 \text{ m/s}$$ (14 points for relative velocity)

$$v_C = \frac{80}{3.6} = 22.22 \text{ m/s}$$

Attach a translating *xy*-frame to the car *C* with the *x*-axis along the normal direction of the truck *T*. The same unit vectors **i**, **j** can be used in both the translating and fixed frames. Thus

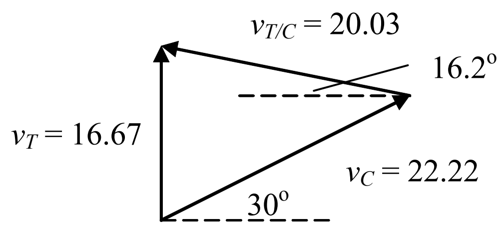
$$\mathbf{v}_T = \mathbf{v}_C + \mathbf{v}_{T/C}$$

$$\Rightarrow \quad 16.67\mathbf{j} = 22.22\cos 30°\mathbf{i} + 22.22\sin 30°\mathbf{j} + \mathbf{v}_{T/C}$$

$$\Rightarrow \quad \mathbf{v}_{T/C} = -19.24\mathbf{i} + 5.56\mathbf{j} \text{ m/s}$$ (3 points for answer)

The solution may also be obtained graphically from the vector diagram of velocities. In comparison, the absolute velocity of truck *T* is

$$\mathbf{v}_T = 16.67\mathbf{j}$$



Similarly, (14 points for relative acceleration)

$$a_T = \frac{v_T^2}{\rho} = \frac{16.67^2}{110} = 2.53 \text{ m/s}^2$$

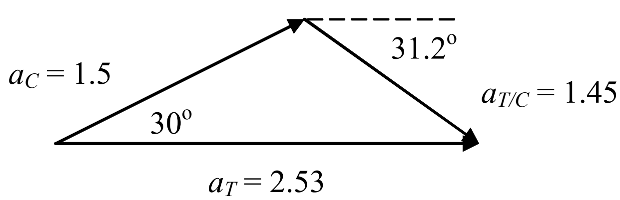
$$a_C = 1.5 \text{ m/s}^2$$

The acceleration of *T* relative to *C* is given by

$$\mathbf{a}_T = \mathbf{a}_C + \mathbf{a}_{T/C}$$

$$\Rightarrow \quad 2.53\mathbf{i} = 1.5\cos 30°\mathbf{i} + 1.5\sin 30°\mathbf{j} + \mathbf{a}_{T/C}$$

$$\Rightarrow \quad \mathbf{a}_{T/C} = 1.23\mathbf{i} - 0.75\mathbf{j} \text{ m/s}^2$$ (3 points for answer)



(b) (5 points) A coordinate system attached to the truck *T* (with the *y*-axis in the direction of the velocity of *T* for example) is a rotating system. If $\mathbf{v}_{\text{rel}}$ is the velocity of car *C* as observed from truck *T*, then $\mathbf{v}_{\text{rel}}$ depends on the rotation of coordinate system attached to the truck. It can be shown from rigid-body kinematics that

$$\mathbf{v}_{\text{rel}} = \mathbf{v}_C - \mathbf{v}_T - \boldsymbol{\omega} \times \mathbf{r}_{C/T}$$ (this is not expected)

$$\Rightarrow \quad \mathbf{v}_{\text{rel}} \neq \mathbf{v}_C - \mathbf{v}_T = -\mathbf{v}_{T/C}$$ (3 points for answer)

Problem 2. (33 points total)

Attach *xy*-frame to the center of the upper pulley. Locate *A*, *B* by coordinates x_A and y_B. Since $x_A \le 0$,

$$-x_A + 2y_B = L \quad \Rightarrow \quad v_A = 2v_B \qquad \text{(7 points for constraint)}$$

$$\Rightarrow \quad v_A(0) = 2v_B(0) = 6 \text{ ft/sec}$$

For block *A*, (10 points for block *A*)

$$\int \sum F_x dt = \Delta G_x \quad \Rightarrow \quad \int_0^1 (T - \mu_k N)dt = m_A v_A(1) - m_A v_A(0) \quad \longrightarrow$$

$$\Rightarrow \quad T - 0.1(10) = \frac{10}{32.2}[v_A(1) - 6] \qquad (1)$$

For block *B*, (10 points for block *B*)

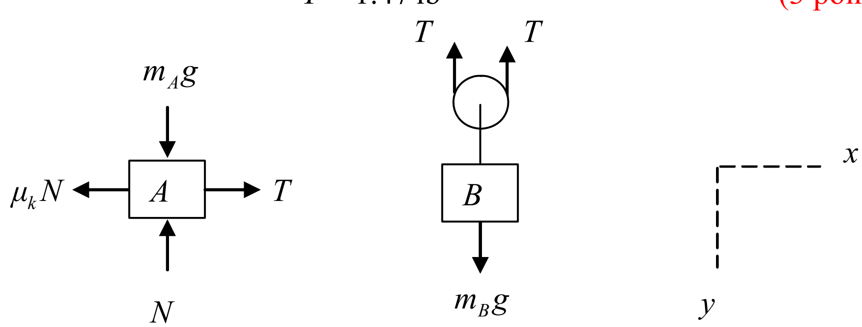
$$\int \sum F_y dt = \Delta G_y \quad \Rightarrow \quad \int_0^1 (-2T + m_B g)dt = m_B v_B(1) - m_B v_B(0) \quad \downarrow$$

$$\Rightarrow \quad -2T + 3 = \frac{3}{32.2}\left[\frac{v_A(1)}{2} - 3\right] \qquad (2)$$

There are 2 unknowns $v_A(1)$, T in 2 equations. Simultaneous solution gives

$$v_A(1) = 7.50 \text{ ft/sec} \quad \longrightarrow \qquad \text{(3 points for answer)}$$

$$T = 1.47 \text{ lb} \qquad \text{(3 points for answer)}$$



Problem 3. (34 points total)

The solution is divided into three parts so that the impact between *A* and *B* can be treated separately.

Part 1. (10 points) Let v_A be the velocity with which the block *A* hits the pan *B*.

$$\Delta T + \Delta V_g = 0 \Rightarrow \quad m_A gh = \frac{1}{2} m_A v_A^2$$

$$\Rightarrow \quad v_A = 6.26 \text{ m/s} \quad \downarrow$$

Part 2. (10 points) Let *v* be the common velocity of the block *A* and pan *B* after the plastic impact. Then

$$\Delta G = 0 \quad \Rightarrow \quad m_A v_A + m_B v_B = (m_A + m_B)v$$

$$\Rightarrow \quad 30(6.26) = (30 + 10)v$$

$$\Rightarrow \quad v = 4.70 \text{ m/s} \quad \downarrow$$

Part 3. (14 points) Let δ be the maximum deflection of the pan measured from its initial level of static equilibrium. The initial compression of the spring is

$$e = m_B g / k = 4.91 \times 10^{-3} \text{ m}$$

For the system consisting of A and B,

$$U_{1-2} = \Delta T = -T_1$$

$$\Rightarrow \quad (m_A + m_B) g\delta - \int_e^{e+\delta} kx\,dx = -\frac{1}{2}(m_A + m_B) v^2$$

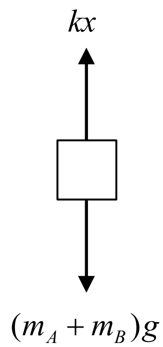
$$\Rightarrow \quad 40(9.81)\delta - 10000[(e+\delta)^2 - e^2] + 20v^2 = 0$$

$$\Rightarrow \quad 10000\delta^2 - 294.2\delta - 441.8 = 0$$

$$\Rightarrow \quad \delta = 0.225 \text{ or } -0.196$$

Take the positive root to write

$$\delta = 225 \text{ mm}$$ (5 points for answer)



Position 1: at reference level

Position 2: δ below reference