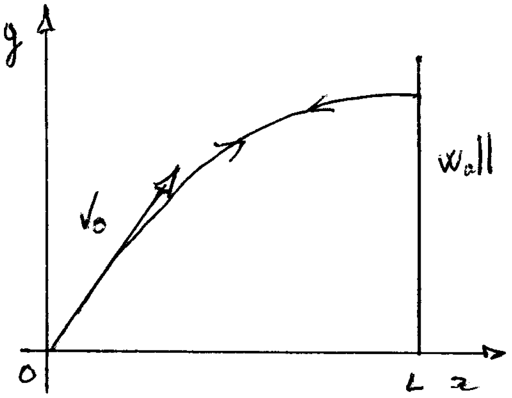
Physic 7A Midterm 1 Problem 1 Lecture 2 & 3

Solo game of catch Solution

a) Sketch of the ball's trajectory



There is no air-friction and the collision is perfectly elastic so the y-component of the velocity must be equal to zero when the ball hits the wall.
The return trajectory will be the same for the throw and for the return.

The equations of the ball motion are:

x - component

$a_x = 0$

$v_x = V_0 \cos\theta$

$x = V_0 . \cos\theta . t$

y - component

$a_y = -g$

$v_y = -g.t + V_0 \sin\theta$

$y = -g\frac{t^2}{2} + V_0 \sin\theta . t$

At time $t = T$, when the ball hits the wall, we have the following relations: for $t=T$ $\begin{cases} v_y = 0 \\ x = L \end{cases}$ $\quad \begin{cases} -gT + V_0 \sin\theta = 0 \\ V_0 \cos\theta . T = L \end{cases}$

If we substitute $T = \frac{L}{V_0 \cos\theta}$ in the first relation, we have the relation between v_0 and θ:

$$\boxed{V_0 = \sqrt{\frac{gL}{\sin\theta \cos\theta}} = \sqrt{\frac{2gL}{\sin 2\theta}}}$$

b) If $V_0 = V_{max}$, we have

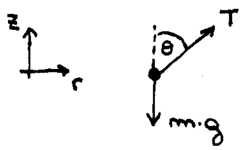
$$V_{max}^2 = \frac{2gL}{\sin 2\theta} \quad \text{so} \quad L_{max} = \max\left(\frac{V_{max}^2 \sin 2\theta}{2g}\right)$$

The maximum of this function is reach for $\theta = \frac{\pi}{4}$

So $$\boxed{\begin{aligned} \theta_{max} &= \frac{\pi}{4} = 45^\circ \\ L_{max} &= \frac{V_{max}^2}{2g} \end{aligned}}$$

$\boxed{P2,\ MT1,\ L2\ \&\ 3}$

a) <u>FBD m</u>



b) $\Sigma F_r: \quad T\cdot\sin(\theta) = m\cdot a_c = m\cdot\frac{V^2}{R} \qquad (1)$

$\Sigma F_z: \quad T\cdot\cos(\theta) - mg = 0 \qquad (2)$

But : $\quad R = L\cdot\sin(\theta) \qquad (3)$

$(2) \rightarrow \quad T = \frac{mg}{\cos(\theta)}$

(1) and $(3) \rightarrow \quad mg\,\tan(\theta) = m\,\frac{V^2}{L\cdot\sin(\theta)} \quad \rightarrow \quad \boxed{V = \sqrt{L\cdot g\cdot Tg(\theta)\cdot\sin(\theta)}}$

Now : $\quad \omega = \frac{V}{R} = \frac{\sqrt{\frac{L\cdot g\cdot\sin^2(\theta)}{\cos(\theta)}}}{L\cdot\sin(\theta)} = \sqrt{\frac{g}{L\cdot\cos(\theta)}}$

$\rightarrow \quad \boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L\cdot\cos(\theta)}{g}}}$

c) $\boxed{T = \frac{mg}{\cos(\theta)}}$

$a_c = \frac{V^2}{R} = \frac{L\cdot g\cdot Tg(\theta)\cdot\sin(\theta)}{L\cdot\sin(\theta)}$

$\rightarrow \quad \boxed{a_c = g\cdot Tg(\theta)}$ in the radial direction

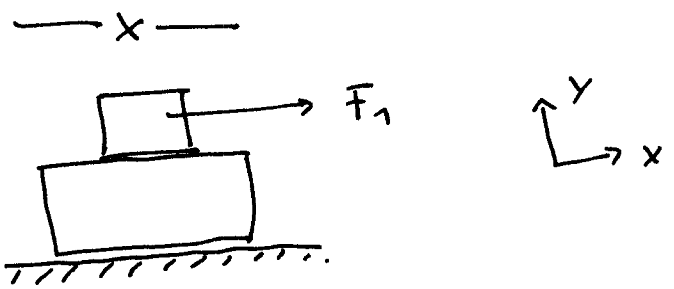
(or pointing toward the center)

Lecture 3, MT 1, Question 3

- For both cases the blocks do not move in the direction perpendicular to the table: the accelerations in that direction are zero.
- The condition that the blocks do not slip is that they have the same velocity parallel to the table: the accelerations in that direction are equal:

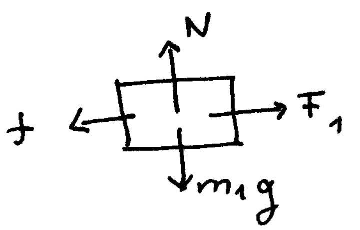
$$a_1 = a_2 = a. \qquad (*)$$

a) Set-up:

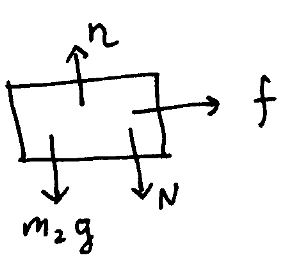


Free-Body Diagrams:

for m_1



for m_2



- here f is static friction force due to surface between blocks.
- N is normal force from interface between blocks
- n is normal force from table.

. note: the reason f points in opposite direction to F_1 on m_1 is to stop/resit the acceleration ~~of~~ caused by F_1. This cause a forward force on m_2, which is needed so that m_2 accelerates forward with m_1

- Neuton 2 for blocks

$$\left.\begin{aligned} F_1 - f &= m_1 a \\ f &= m_2 a \end{aligned}\right\} \text{using equal accelerations } (*)$$

and $N - m_1 g = 0$ (vertical for m_1).

- Hence :

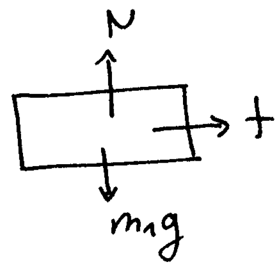
$$F_1 = f\left(1 + \frac{m_1}{m_2}\right)$$

Since friction is $f \leq \mu N$ (static, no-slip)

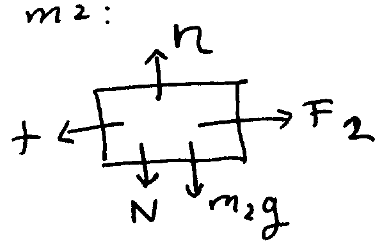
$$\boxed{F_1 \leq \mu m_1 g \left(1 + \frac{m_1}{m_2}\right)}$$

b) FBD :

for m_1



for m_2:



Newton 2:

$$f = m_1 a$$
$$F_2 - f = m_2 a$$
$$N - m_1 g = 0.$$

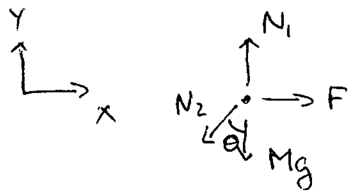
Proceeding as above: $F_2 = f\left(1 + \frac{m_2}{m_1}\right)$

$$\boxed{F_2 \leq \mu m_1 g \left(1 + \frac{m_2}{m_1}\right)}$$

Physics 7A, Sections 2 and 3, Midterm 1

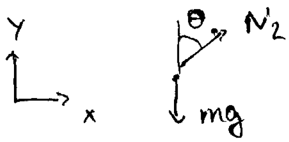
Problem 4

a) FBD of mass M



$$\begin{cases} \sum F_x = F - N_2 \sin\theta = Ma & (1) \\ \sum F_y = N_1 - Mg - N_2 \cos\theta = 0 & (2) \end{cases}$$

FBD of mass m



$$\begin{cases} \sum F_x = N_2 \sin\theta = ma & (3) \\ \sum F_y = N_2 \cos\theta - mg = 0 & (4) \end{cases}$$

Equation (4) gives us $N_2 = \dfrac{mg}{\cos\theta}$

Equation (3) becomes $a = \dfrac{N_2 \sin\theta}{m} = \dfrac{mg \sin\theta}{m\cos\theta} \longrightarrow a = g\tan\theta$

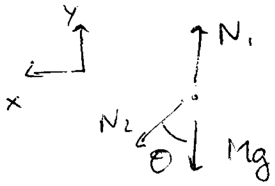
We substitute N_2 and a in equation (1)

$$F = N_2 \sin\theta + Ma$$

$$= \frac{mg\sin\theta}{\cos\theta} + Mg\tan\theta$$

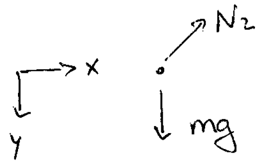
$$\boxed{F = (m+M)g\tan\theta}$$

b) FBD of mass M



$$\begin{cases} \Sigma F_x = N_2 \sin\theta = M a_{M,x} & (5) \\ \Sigma F_y = N_1 - Mg - N_2 \cos\theta = M a_{M,y} & (6) \end{cases}$$

FBD of mass m



$$\begin{cases} \Sigma F_x = N_2 \sin\theta = m a_{m,x} & (7) \\ \Sigma F_y = mg - N_2 \cos\theta = m a_{m,y} & (8) \end{cases}$$

The mass M moves horizontally so its vertical acceleration is zero,

$\boxed{a_{M,y} = 0.}$

From equations (5) and (7) we get $M a_{M,x} = m a_{m,x}$

$$\boxed{a_{M,x} = \frac{m}{M} a_{m,x}} \quad (*)$$

In our problem there is a condition between the two accelerations of the blocks so $\tan\theta = \dfrac{a_{m,y}}{a_{m,x} + a_{M,x}}$. By plugging in $(*)$ we obtain $\tan\theta = \dfrac{a_{m,y}}{a_{m,x}\left(1 + \frac{m}{M}\right)}$

$$\boxed{a_{m,y} = \tan\theta . a_{m,x} . \left(1 + \frac{m}{M}\right)} \quad (**)$$

Equation (7) gives $N_2 = \dfrac{m a_{m,x}}{\sin\theta}$, that we plug in (8)

$$m a_{m,y} = mg - m a_{m,x} \frac{\cos\theta}{\sin\theta}$$

$$\boxed{a_{m,y} = g - a_{m,x} \frac{1}{\tan\theta}} \quad (***)$$

From (**) and (***) we obtain

$$\tan\theta \cdot a_{m,x} \cdot \left(1 + \frac{m}{M}\right) = g - \frac{a_{m,x}}{\tan\theta}$$

$$a_{m,x} \tan^2\theta \cdot \left(1 + \frac{m}{M}\right) + a_{m,x} = g\tan\theta$$

$$a_{m,x}\left[1 + \tan^2\theta\left(1 + \frac{m}{M}\right)\right] = g\tan\theta$$

$$\boxed{a_{m,x} = \frac{g\tan\theta}{1 + \tan^2\theta\left(1 + \frac{m}{M}\right)}}$$

From (*) $\boxed{a_{M,x} = \frac{m}{M} \cdot \frac{g\tan\theta}{1 + \tan^2\theta\left(1 + \frac{m}{M}\right)}}$

From (**) $\boxed{a_{m,y} = \frac{\left(1 + \frac{m}{M}\right) g\tan^2\theta}{1 + \tan^2\theta\left(1 + \frac{m}{M}\right)}}$