Prob-1

(a) By ideal gas law 2pt's

we have $P \cdot V = N k_B T$. $\Rightarrow$ $N = \frac{PV}{k_B T} = \frac{P(a^2 h)}{k_B T}$ 3pt's

(−1 pt if you only consider N_2 and get $f \cdot \frac{P(a^2h)}{k_B T}$)

(b) after increasing T by ΔT. we get new dimensions of the house to be $\begin{cases} a' = a(1+\alpha\Delta T) \\ h' = h(1+\alpha\Delta T) \end{cases}$ $\Rightarrow$ $V' = a'^2 \cdot h' = a^2 h (1+\alpha\Delta T)^3$

$= \underset{\Downarrow}{a^2 h} (1 + 3\alpha\Delta T + 3(\alpha\Delta T)^2 + (\alpha\Delta T)^3)$

While the linear expansion formula is correct up to linear terms in ΔT. we should really throw away all the higher order terms

$\Rightarrow V' = V(1+3\alpha\Delta T)$. and $\frac{\Delta V}{V} = 3\alpha\Delta T$ 3pt. , 5pt

(2pt for the explanation if you don't get the final result right!)

(c) One has to realize that because the house is not air tight after a while it reaches equilibrium, i.e. $\begin{cases} P_{inside} = P_{outside} = P \\ T_{inside} = T_{outside} = T' = (T+\Delta T) \end{cases}$

(if you don't get the final result right, but you point out the two concepts here, you get partial credit 4pt's/10pts)

$\Rightarrow$ $PV' = N' k_B T'$ 2pt's

$\Rightarrow$ $\rho' = \frac{N'}{V'} = \frac{P}{k_B T'} = \frac{P}{k_B (T+\Delta T)}$ 2pt's.

when $\frac{\Delta T}{T}$ is small. one can do the approximation

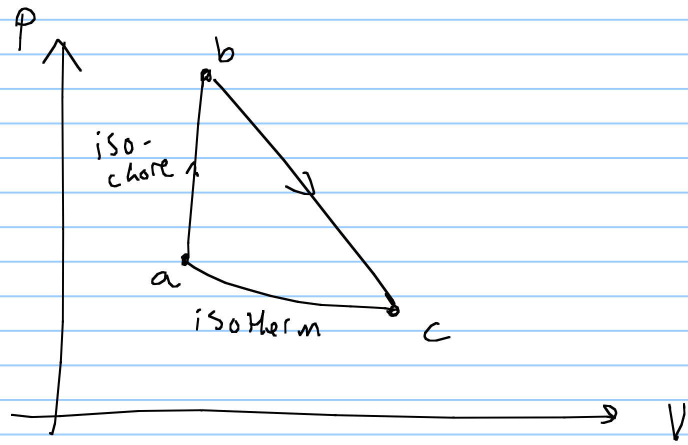
$\rho' = \frac{P}{k_B T(1+\frac{\Delta T}{T})} \sim \frac{P}{k_B T}(1 - \frac{\Delta T}{T})$ 2pt's.

while $\rho_0 = \frac{N}{V} = \frac{P}{k_B T}$ 2pt $\Rightarrow$ $\frac{\Delta \rho}{\rho_0} = -\frac{\Delta T}{T}$ 2pt

Phys 7B Midterm #1, Problem 2 solution

Note Title 2/25/2012

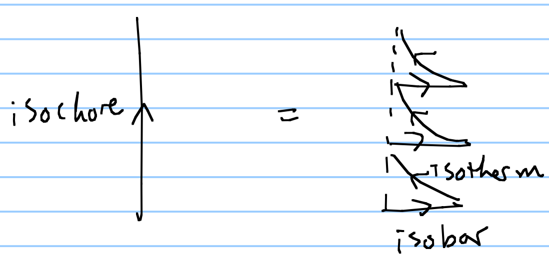
(2)



a) ab: Step 1: add scoop of hot water to raise temp by (.1°C)

Step 2: add small weight necessary to keep V constant.

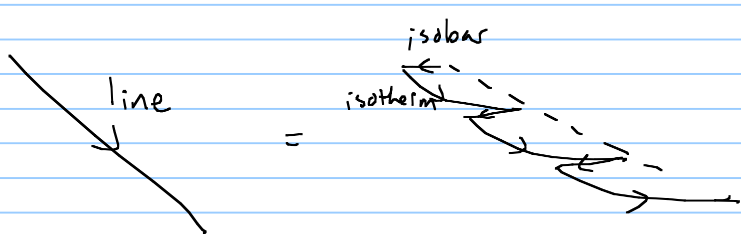
* repeat until $P = P_2$



b) bc: step 1: add a scoop of cold water (since $T_b > T_c$) for (-.1°C)

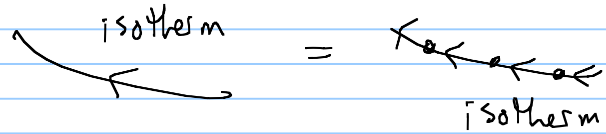
step 2: remove weights such that ($\Delta P/\Delta V$ = desired slope)

* repeat until $V = V_2$

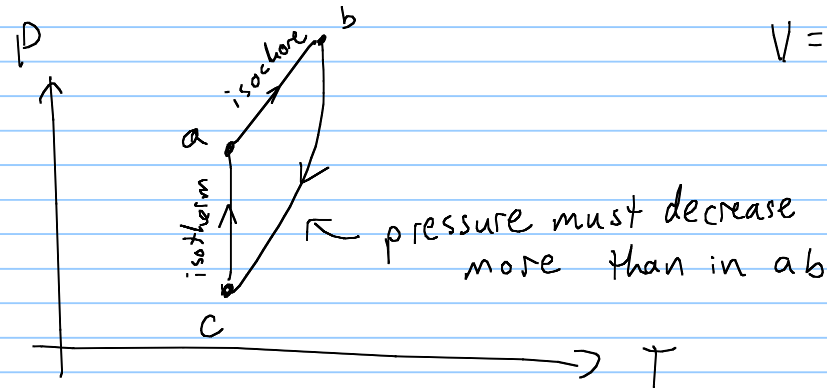


c) ca: step 1: add a small weight to increase P by .01 atm

* repeat until $P = P_1$



On P–T diagram (our independent variables):



$V = \frac{T}{P}$

Problem 3

Grading notes:

No points for unused formulas

If you used ΔSideal from your cheat sheet, there was a maximum of 4 points total. The point of the problem was to work out the entropy integral.

It was okay if you used any correct path over which to integrate S because S is a state variable.

Solve for : Entropy change from B → C

$$dS = \frac{dQ}{T}$$

Maximum 5 pts. for setting up entropy integral and using the integral. No points for just the formula.

$$\Delta S = \int \frac{1}{T}\, dQ$$

Maximum 5 pts.: Second Law. You must state and use the law correctly for credit.

Maximum 5pts.: both entropy terms are algebraically correct

(If the 2nd law was not used in one place and path was split up into different phases (isochoric, isobaric, etc) then there was 5 pts. max for each term.)

$$\begin{aligned} \text{Second law: } Q &= \Delta E + W \\ dQ &= dE + dW \\ &= \frac{d}{2} NkdT + PdV \end{aligned}$$

$$\Delta S = \int \frac{(d/2)\, Nk}{T}\, dT + \int \frac{P}{T}\, dV$$

$$= \frac{d}{2} Nk * \ln\left(\frac{Tc}{Tb}\right) + Nk * \ln\left(\frac{Vc}{Vb}\right)$$

Max 3pts. : Plug in temperatures and volumes

$$Tc = Ta = \frac{P1 * V1}{Nk} \quad \text{and} \quad Tb = \frac{P2 * V1}{Nk}$$

$$Vc = V2 \quad \text{and} \quad Vb = V1$$

Max 2pts: Carry out all algebra to finish, including signs of terms.

$$Nk = nR = R \quad (\text{for 1 mol})$$

$$d = 3 \quad (\text{for monatomic})$$

Answer:

$$\Delta Sbc = \frac{3}{2} R * \ln\left(\frac{P1}{P2}\right) + R * \ln\left(\frac{V2}{V1}\right)$$

④ ⓐ Since the system is isolated

$$Q_{cup} + Q_{H_2O} = 0$$

$$m_1 c_1 (T_f - T_1) + m_2 c_2 (T_f - T_2) = 0$$

$$(m_1 c_1 + m_2 c_2) T_f = m_1 c_1 T_1 + m_2 c_2 T_2$$

$$\boxed{T_f = \frac{m_1 c_1 T_1 + m_2 c_2 T_2}{m_1 c_1 + m_2 c_2}}$$

ⓑ Since the temperature is changing, we must use

$$\Delta S = \int_{initial}^{final} dS = \int \frac{dQ}{T} = \int \frac{mc\,dT}{T}$$

$$\Delta S_1 = \int_{T_1}^{T_f} \frac{m_1 c_1 dT}{T} = m_1 c_1 \ln\left(\frac{T_f}{T_1}\right)$$

$$\Delta S_2 = \int_{T_2}^{T_f} \frac{m_2 c_2 dT}{T} = m_2 c_2 \ln\left(\frac{T_f}{T_2}\right)$$

$$\boxed{\Delta S_{total} = \Delta S_1 + \Delta S_2 = m_1 c_1 \ln\left(\frac{T_f}{T_1}\right) + m_2 c_2 \ln\left(\frac{T_f}{T_2}\right)}$$

($\Delta S_{environment} = 0$ because the system is isolated)

5. a. (5 pts) Write equations for each rod, noting that the temperature of each rod at the junction is the same.

$$r_1 = \frac{Ak_1}{L_1}(T_H - T_M) \quad r_2 = \frac{Ak_2}{L_2}(T_M - T_L)$$

At steady-state, the rates will be the same, $r_1 = r_2$. Knowing this, we can solve for T_M.

$$\frac{Ak_1}{L_1}(T_H - T_M) = \frac{Ak_2}{L_2}(T_M - T_L)$$

$$\frac{k_1}{L_1}T_H + \frac{k_2}{L_2}T_L = \left(\frac{k_1}{L_1} + \frac{k_2}{L_2}\right)T_M$$

$$T_M = \frac{\frac{k_1}{L_1}T_H + \frac{k_2}{L_2}T_L}{\frac{k_1}{L_1} + \frac{k_2}{L_2}}$$

b. (5 pts) For this part, we know $T_M = 50\ °C$. I will also plug in $T_H = 100\ °C$ and $T_L = 0\ °C$.

$$50\ °C = \frac{\frac{k_1}{L_1}100\ °C}{\frac{k_1}{L_1} + \frac{k_2}{L_2}}$$

$$\frac{1}{2} = \frac{1}{1 + \frac{L_1}{L_2}\frac{k_2}{k_1}} \rightarrow \frac{L_1}{L_2}\frac{k_2}{k_1} = 1 \rightarrow L_2 = \frac{k_2}{k_1}L_1$$

c. (5 pts) Calculate the rates, r_1 and r_2. Plug in our knowledge of T_M from part a.

$$r_1 = \frac{Ak_1}{L_1}(T_H - T_M) = \frac{Ak_1}{L_1}\left(T_H\frac{\frac{k_1}{L_1} + \frac{k_2}{L_2}}{\frac{k_1}{L_1} + \frac{k_2}{L_2}} - \frac{\frac{k_1}{L_1}T_H + \frac{k_2}{L_2}T_L}{\frac{k_1}{L_1} + \frac{k_2}{L_2}}\right) = \frac{Ak_1}{L_1}\frac{T_H\frac{k_2}{L_2} - T_L\frac{k_2}{L_2}}{\frac{k_1}{L_1} + \frac{k_2}{L_2}} = A\frac{\frac{k_1}{L_1}\frac{k_2}{L_2}}{\frac{k_1}{L_1} + \frac{k_2}{L_2}}(T_H - T_L)$$

$$= A\frac{1}{\frac{L_1}{k_1} + \frac{L_2}{k_2}}(T_H - T_L)$$

Of course, r_2 is the same because we are at equilibrium.

d. (5 pts) If all linear dimensions double, then the length doubles, $L \rightarrow 2L$, and the cross-sectional area quadruples, $A \rightarrow 2^2A$. Looking at the conduction formula makes it clear that the flow of heat doubles (for both r_1 and r_2). It is helpful to note that the temperature at the midpoint does not change, see the answer to part a. above, letting $L_1 \rightarrow 2L_1$ and $L_2 \rightarrow 2L_2$.

$$H = \frac{Ak}{L}(T_2 - T_1) \rightarrow \frac{4Ak}{2L}(T_2 - T_1) = 2\frac{Ak}{L}(T_2 - T_1) = 2H$$