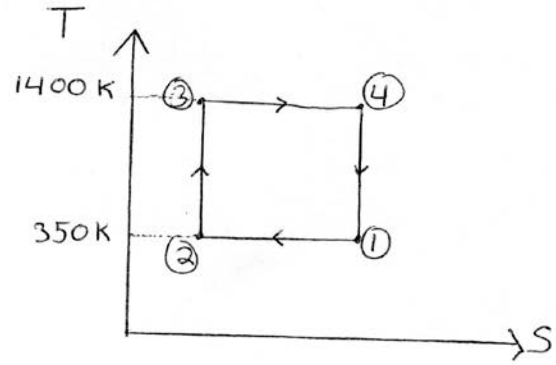
Problem 1

a. (10 points)



b. (15 points)

For an ideal gas with constant specific heats, we can use eqn 7-34:

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$$

Notice that this is specific entropy, so

$$S_2 - S_1 = m(s_2 - s_1) = m\left(c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)\right)$$

Process ① - ②

$$T_1 = T_2 \text{ (isothermal)} \Rightarrow \ln\left(\frac{T_2}{T_1}\right) = 0$$

To find R, use eqn 4-29 (for ideal gas)

$$c_p = c_v + R$$

$$R = c_p - c_v = 1.005\ \text{KJ/KgK} - 0.718\ \text{KJ/KgK}$$

$$R = 0.287\ \text{KJ/KgK}$$

$$S_2 - S_1 = m\left(0 - R\ln\left(\frac{P_2}{P_1}\right)\right)$$

$$S_2 - S_1 = 1\text{Kg}\left(0 - (0.287\ \tfrac{\text{KJ}}{\text{KgK}})\ln\left(\frac{500\ \text{KPa}}{100\ \text{KPa}}\right)\right)$$

$$\boxed{S_2 - S_1 = -0.4619\ \text{KJ/K}}$$

Process ③→④

$$S_4 - S_3 = -(S_2 - S_1) = \boxed{0.4619\ \text{KJ/K}}$$

C. (15 points)

Method 1:

For internally reversible, isothermal process, use eqn 7-18:

$$\Delta Q = T_0 \Delta S$$

Process ① → ②

$$Q_{12} = T_1 (S_2 - S_1)$$

$$Q_{12} = 350\,K(-0.4619\ kJ/K)$$

$$\boxed{Q_{12} = -161.665\ kJ}$$

Process ③ → ④

$$Q_{34} = T_3 (S_4 - S_3)$$

$$Q_{34} = 1400\,K(0.4619\ kJ/K)$$

$$\boxed{Q_{34} = 646.66\ kJ}$$

Method 2:

$$Q_{12} - W_{12} = \cancel{U_{12}}^{\,0}$$

$$U_{12} = c_v (T_2 - T_1) = c_v (350K - 350K) = 0$$

$$Q_{12} = W_{12}$$

$$W_{12} = \int_1^2 P\,dV$$

$$PV = mRT$$

$$P = mRT/V$$

$$W_{12} = mRT_1 \int \frac{dV}{V}$$

$$W_{12} = mRT_1 \ln\left(\frac{v_2}{v_1}\right)$$

To find $\frac{v_2}{v_1}$:

$$P_1 v_1 = RT_1$$

$$P_2 v_2 = RT_2 = RT_1 \quad \text{since } T_1 = T_2$$

$$P_1 v_1 = P_2 v_2$$

$\frac{v_2}{v_1} = \frac{P_1}{P_2} \Rightarrow v_1 = v_2\left(\frac{P_2}{P_1}\right)$ is used on the next page

$$W_{12} = mRT_1 \ln\left(\frac{P_1}{P_2}\right)$$

$$W_{12} = (1\,kg)(0.287\ kJ/kg\,K)(350K)\ln\left(\frac{100\ kPa}{500\ kPa}\right)$$

$$W_{12} = -161.67\ kJ$$

$$\boxed{Q_{12} = -161.67\ kJ}$$

Method 2 continued:

Similar approach to process ③→④

$Q_{34} = W_{34}$

$W_{34} = m R T_3 \ln\left(\frac{v_4}{v_3}\right)$

To find $\frac{v_4}{v_3}$:

isentropic relation for ideal gas:

$$P_2 v_2^{\gamma} = P_3 v_3^{\gamma}$$

$$P_3 = P_2 \left(\frac{v_2}{v_3}\right)^{\gamma}$$

$$P_3 v_3 = R T_3$$

$$P_2 \left(\frac{v_2}{v_3}\right)^{\gamma} v_3 = R T_3$$

$$v_3 = \left(\frac{R T_3}{P_2 v_2^{\gamma}}\right)^{1/(1-\gamma)}$$

isentropic relation for ideal gas (process ①→④):

$$P_1 v_1^{\gamma} = P_4 v_4^{\gamma}$$

$$P_4 = P_1 \left(\frac{v_1}{v_4}\right)^{\gamma}$$

$$P_4 v_4 = R T_4$$

$$P_1 \left(\frac{v_1}{v_4}\right)^{\gamma} v_4 = R T_4$$

$$v_4 = \left(\frac{R T_4}{P_1 v_1^{\gamma}}\right)^{1/(1-\gamma)}$$

$T_4 = T_3$ (isothermal)

$$v_1 = v_2 \left(\frac{P_2}{P_1}\right)$$

$$v_4 = \left(\frac{R T_3}{P_2 v_2^{\gamma}}\right)^{1/(1-\gamma)} \frac{P_2}{P_1}$$

$$\frac{v_4}{v_3} = \frac{\left(\frac{R T_3}{P_2 v_2^{\gamma}}\right)^{1/(1-\gamma)} \frac{P_2}{P_1}}{\left(\frac{R T_3}{P_2 v_2^{\gamma}}\right)^{1/(1-\gamma)}} = \frac{P_2}{P_1}$$

$W_{34} = m R T_3 \ln\left(\frac{P_2}{P_1}\right)$

$W_{34} = (1\text{ kg})(0.287\text{ kJ/kg K})(1400\text{ K}) \ln\left(\frac{500\text{ kPa}}{100\text{ kPa}}\right)$

$W_{34} = 646.67\text{ kJ}$

$$\boxed{Q_{34} = 646.67\text{ kJ}}$$

Differences in final answers in methods 1 and 2 are do to rounding ΔS in method 1.

d. (10 points)

Method 1:

$$Q - W = \cancel{\Delta U}^{\,0}$$

$$Q = W$$

$$W = Q_{12} + Q_{34}$$

$$W = (-161.665 + 646.66)\,\text{kJ}$$

$$\boxed{W = 484.995\ \text{kJ}}$$

Method 2:

$$\eta_{th} = \frac{W_{net,out}}{Q_{in}} \leftarrow \text{general}$$

$$\eta_{carnot} = 1 - \frac{T_L}{T_H} \leftarrow \text{Carnot}$$

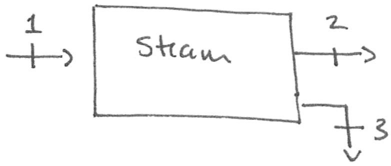
$$\frac{W_{net,out}}{Q_{in}} = 1 - \frac{T_L}{T_H}$$

$$W_{net,out} = Q_{in}\left(1 - \frac{T_L}{T_H}\right) = Q_{34}\left(1 - \frac{T_L}{T_H}\right)$$

$$W_{net,out} = 646.66\,\text{kJ}\left(1 - \frac{350\,\text{K}}{1400\,\text{K}}\right)$$

$$\boxed{W_{net,out} = 484.995\,\text{kJ}}$$

ME 40 Exam 1 Problem 2



$P_1 = 2.0$ MPa
$T_1 = 250\,°C$
$\dot{m}_1 = 0.60$ kg/s

$\left.\begin{array}{l} \\ \\ \end{array}\right\}$ $h_1 = 2903.3$ kJ/kg, $s_1 = 6.5475$ kJ/kg-K

$P_3 = 0.10$ MPa

$P_2 = 0.30$ MPa
$x = 0.4$
$\dot{m}_2 = 0.10$ kg/s

$h_2 = h_f + x h_{fg} = 561.4 + (0.4)\,2163.9 = 1426.8$ kJ/kg
$s_2 = s_f + x s_{fg} = 1.6717 + (0.4)\,5.32 = 3.7997$ kJ/kg-K

$\dot{Q} \simeq 0$, KE $\approx$ PE ≈ 0

(a) $\dot{W}_{out} = 350$ kW

find h_3 and T_3

<u>Consv. of mass</u>

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3 \Rightarrow \dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 0.6 - 0.1 = 0.5 \text{ kg/s}$$

<u>Consv. of Energy (SSSF)</u>

$$-\dot{W}_{out} + \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 = 0$$

$$\dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{W}_{out} = \dot{m}_3 h_3$$

$$h_3 = \frac{\dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{W}_{out}}{\dot{m}_3} = \frac{(0.6)(2903.23) - (0.1)(1426.8) - 350}{0.5}$$

$\boxed{h_3 = 2498.51 \text{ kJ/kg}}$ from tables $\rightarrow$ $\boxed{T_3 = 99.6\,°C}$

Exam 1 Problem 2 continued...

(b) find $\dot{W}_{max,out}$

Must be reversible $\rightarrow$ $\dot{S}_{gen} = 0$

Entropy Balance:

$$\dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{m}_3 s_3 = 0$$

$$s_3 = \frac{\dot{m}_1 s_1 - \dot{m}_2 s_2}{\dot{m}_3} = \frac{(0.6)(6.5475) - (0.1)(3.7997)}{0.5}$$

$s_3 = 7.09694$ kJ/kg-K

at 0.1 MPa $\rightarrow$ $s_f = 1.3028$, $s_g = 7.3589$ $\Rightarrow$ Saturated

$$x_3 = \frac{s_3 - s_f}{s_{fg}} = 0.957$$

$$h_{3,rev} = h_f + x_3 h_{fg} = (417.5) + 0.957(2257.5) = 2577.9 \text{ kJ/kg}$$

$$\dot{W}_{max,out} = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_{3,rev}$$
$$= (0.6)(2903.2) - (0.1)(1426.8) - (0.5)(2577.9)$$

$$\boxed{\dot{W}_{max,out} = 310.6 \text{ kW}}$$

(c) Not accurate becaus $\dot{W}_{max,out} < \dot{W}_{out}$.
Not thermodynamically possible. It violates the 2nd Law.