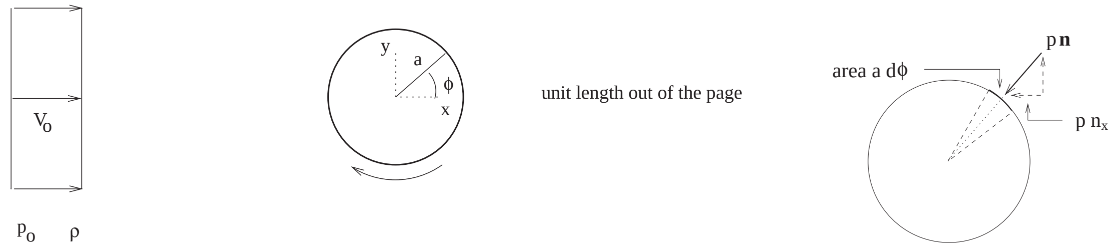
Overall Mean: 154.17, SD: 39.24

UNIVERSITY OF CALIFORNIA, BERKELEY
MECHANICAL ENGINEERING
ME106 Fluid Mechanics
1st Test, S12 *Prof S. Morris*

Problem 1
Mean: 44.96
SD: 20.81

SOLUTIONS

1. (65) Far from the spinning *cylinder*, the air of density ρ has uniform velocity $V_o\mathbf{i}$ and pressure p_o. On the cylinder, the pressure is given as a function of angle ϕ by $p(\phi) - p_o = -4\rho V_0^2(J + \sin\phi)^2$; J is a given constant. The aim is to find the component of the resultant pressure force acting *parallel* to the free stream $V_O\mathbf{i}$.



(a) Derive the expression giving F_x as an integral of $p(\phi)$ with respect to ϕ.

(b) Evaluate your integral to determine F_x.

(c) On a single sketch, show $J + \sin\phi$ and $(J + \sin\phi)^2$ as functions of ϕ; then interpret your answer to part (b) using that sketch. For full credit, all curves and axes on your sketch must be clearly labelled.

Given: $n(\cos\phi)(\sin^{n-1}\phi) = \frac{\mathrm{d}}{\mathrm{d}\phi}\big(\sin^n\phi\big)$.

(a) The right hand figure shows the elementary area $a\mathrm{d}\phi$; it is chosen so that the stress vector $-p\mathbf{n}$ is constant on it.

Resolving the stress vector into its Cartesian components, we see that the x–component of force (per unit length) exerted by the stream on the elementary area is given by

$$-pn_x a\mathrm{d}\phi = -ap\cos\phi\,\mathrm{d}\phi.$$

The resultant forces parallel to the free stream is given by

$$F_x = -a\int_0^{2\pi} p\cos\phi\,\mathrm{d}\phi.$$

(b) Without approximation,

$$F_x = -a\int_0^{2\pi}(p - p_0)\cos\phi\,\mathrm{d}\phi - a\int_0^{2\pi} p_0\cos\phi\,\mathrm{d}\phi.$$

Setting $n = 1$ in the datum, we see that the second integral vanishes; a uniform pressure p_0 acting over the entire surface of body exerts no resultant force.

Substituting for $p - p_0$ in the remaining integral, we obtain

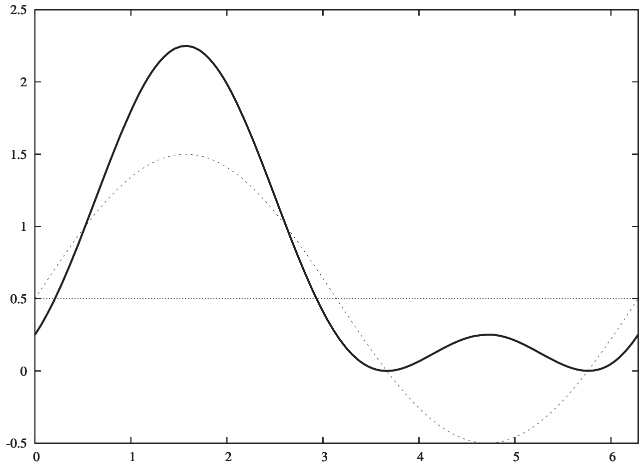
$$\begin{aligned} F_x =& 4\rho V_0^2 a\int_0^{2\pi}(J + \sin\phi)^2\,\cos\phi\,\mathrm{d}\phi, \\ =& 4\rho V_0^2 a\int_0^{2\pi}\frac{\mathrm{d}}{\mathrm{d}\phi}\tfrac{1}{3}(J + \sin\phi)^3\,\mathrm{d}\phi \\ =& \tfrac{4}{3}\rho V_0^2 a[(J + \sin\phi)^3]_0^{2\pi}, = 0. \end{aligned}$$

(The second line follows from the chain rule, and the datum with $n = 3$.)

Conclusion: $F_x = 0$. For this pressure distribution, there is no drag; the component of force parallel to the free stream vanishes identically.

(c) Note: to explain why $F_x = 0$ for the given pressure distribution, only a careful freehand sketch is needed. However, to facilitate making the figure electronically, I have graphed the functions quantitatively for $J = \frac{1}{2}$; the horizontal broken line shows this value of J.

As the broken curve and solid curve respectively, the figure shows $J + \sin\phi$ and $(J + \sin\phi)^2$ as functions of ϕ. To be useful, the sketch *must indicate* that $p - p_0$ is an *even* function of both $\phi - \frac{\pi}{2}$, and of $\phi - \frac{3}{2}\pi$.



In the figure, the upper half cylinder corresponds to $0 < \phi < \pi$. Because $p - p_0$ is an *even* function of $\phi - \frac{\pi}{2}$, we see that the pressure forces contributed by the front $(0 < \phi < \frac{\pi}{2})$ and back $(< \frac{\pi}{2} < \phi < \pi)$ cancel exactly. The upper half cylinder experiences no drag.

The same is true of the lower half cylinder; it corresponds to $\pi < \phi < 2\pi$. Because $p - p_0$ is an *even* function of $\phi - \frac{3}{2}\pi$, we again see that the pressure forces contributed by the front $(\frac{3}{2}\pi < \phi < 2\pi)$ and back $(\pi < \phi < \frac{3}{2}\pi)$ cancel exactly. The lower half cylinder experiences no drag. Owing to this symmetry of the pressure distribution, $F_x = 0$.

Grade Keys,

Problem 1

(a) (30) Answer should be correct but, if you show a vector triangle with the idea of nx but have incorrect trigonometry giving nx ≠cosφ then (10/30), otherwise no partial credit.

(b) (20) If the Integration is done correctly, but no conclusion is reached (Fx = 0), give (10/20), otherwise no partial credit.

(c) (15) Sketch shows symmetry for 0 to 1/2π & 1/2π to π and π to 3/2π & 3/2π to 2π at both plots with explain of conclusion (b), otherwise no partial credit.

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FULL CREDIT FOR CORRECT FINAL RESULT WITH COHERENT ARGUMENT

Problem 2

Mean: 44.26

SD: 18.31

2. (65) (a) Write the formula for the material derivative $\frac{\mathrm{d}f}{\mathrm{d}t}$ of an arbitrary function $f(x,y,z,t)$.

(b) Using the formula from part (a), evaluate $\frac{\mathrm{d}x}{\mathrm{d}t}$; to receive credit, you must explain briefly the values you give to each term in the expression for $\frac{\mathrm{d}x}{\mathrm{d}t}$.

(c) For the flow given by $\mathbf{V} = (Kx + Ly)\mathbf{i} + (Lx - Ky)\mathbf{j}$, find the fluid acceleration $\mathbf{a}$. (Hint: $\mathbf{a} \parallel \mathbf{r}$.)

(a)

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} + w\frac{\partial f}{\partial z}, = \frac{\partial f}{\partial t} + (\mathbf{V}\cdot\nabla)f \qquad (1a,b)$$

(Either form is acceptable.)

(b)

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\partial x}{\partial t} + u\frac{\partial x}{\partial x} + v\frac{\partial x}{\partial y} + w\frac{\partial x}{\partial z}, = 0 + u.1 + v.0 + w.0, = u \qquad (2a,b,c)$$

Explanation: in each partial derivative, all independent variables but one are fixed; consequently

$$\frac{\partial x}{\partial t} = \left(\frac{\partial x}{\partial t}\right)_{(x,y,z)\text{ fixed}} = 0;$$

similarly $\frac{\partial x}{\partial y} = 0 = \frac{\partial x}{\partial z}$, but

$$\frac{\partial x}{\partial x} = \left(\frac{\partial x}{\partial x}\right)_{(y,z,t)\text{ fixed}} = 1.$$

(c) *Method 1: form the total derivative directly.*

With $u = Kx + Ly$ and $v = Lx - Ky$,

$$\mathbf{a} = \frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} = \left(K\frac{\mathrm{d}x}{\mathrm{d}t} + L\frac{\mathrm{d}y}{\mathrm{d}t}\right)\mathbf{i} + \left(L\frac{\mathrm{d}x}{\mathrm{d}t} - K\frac{\mathrm{d}y}{\mathrm{d}t}\right)\mathbf{j},$$

because for Cartesian coordinates, the unit vectors $\{\mathbf{i},\mathbf{j},\mathbf{k}\}$ have constant direction: $0 = \frac{\mathrm{d}\mathbf{i}}{\mathrm{d}t} = \frac{\mathrm{d}\mathbf{j}}{\mathrm{d}t} = \frac{\mathrm{d}\mathbf{k}}{\mathrm{d}t}$.

Because $\frac{\mathrm{d}x}{\mathrm{d}t} = u$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = v$,

$$\begin{aligned}\mathbf{a} =& (Ku + Lv)\mathbf{i} + (Lu - Kv)\mathbf{j},\\ =& \{K(Kx + Ly) + L(Lx - Ky)\}\mathbf{i} + \{L(Kx + Ly) - K(Lx - Ky)\}\mathbf{j},\\ =& (K^2 + L^2)(x\mathbf{i} + y\mathbf{j}).\end{aligned} \qquad (3a,b,c)$$

(3b) follows from (3a) by substituting for u and v; (3c) follows by simplifying (3b).

Method 2: use the general form for $\mathbf{a}$.

Because the flow is steady $\partial\mathbf{V}/\partial t = 0$, so

$$\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} = (\mathbf{V}\cdot\nabla)\mathbf{V}, = u\frac{\partial\mathbf{V}}{\partial x} + v\frac{\partial\mathbf{V}}{\partial y} + w\frac{\partial f}{\partial z} \qquad (4)$$

Because $\mathbf{V} = (Kx + Ly)\mathbf{i} + (Lx - Ky)\mathbf{j}$,

$$\frac{\partial\mathbf{V}}{\partial x} = K\mathbf{i} + L\mathbf{j}, \quad \frac{\partial\mathbf{V}}{\partial y} = L\mathbf{i} - Ky\mathbf{j}. \qquad (5a,b)$$

Substituting (5) into (4), the substituting for u and v, then simplifying, we again obtain (3c).

By either method,

$$\mathbf{a} = (K^2 + L^2)\mathbf{r}, \qquad (6)$$

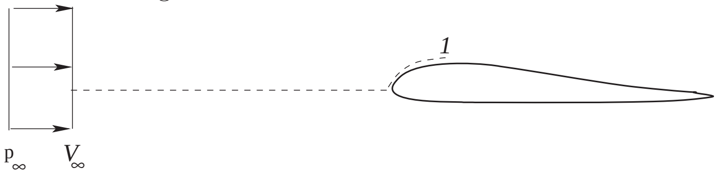
where the position vector $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$.

Problem 3

Mean: 64.82

SD: 11.71

3. (70) At point 1 on the surface of the airfoil, the pressure p is given by $(p - p_\infty)/(\frac{1}{2}\rho V_\infty^2) = -3$. Find the ratio of the flow speed at that point to V_∞. To receive credit, you must explain your logic; a formula and a number is not enough.



Applying the Bernoulli equation along the stagnation streamline (broken line) from infinity to point 1, we have:

$$p_\infty + \tfrac{1}{2}\rho V_\infty^2 = p_1 + \tfrac{1}{2}\rho V_1^2.$$

(We are taking taking changes in potential energy to be negligibly small; this is good approximation if $V_\infty^2 \gg g\Delta z$.)

Solving for $\frac{V_1^2}{V_\infty^2}$, we obtain

$$\frac{V_1^2}{V_\infty^2} = 1 + \frac{p_\infty - p_1}{\frac{1}{2}\rho V_\infty^2}.$$

For the data given, $V_1/V_\infty = 2$. The speed at point 1 is twice that of the free stream.

The streamline must be identified.

Grade Keys,

Problem 2

(a) (10) df/dt=∂f/∂t+(dx/dt)(∂f/∂x)+(dy/dt)(∂f/∂y)+(dz/dt)(∂f/∂z) is also acceptable.

(b) (20) For credit, Final result must be present and all 4 terms must be correctly accounted for.

(c) (35) Final result (20) pts, Set up correctly (15) pts

Problem 3

(a) (15) Bernoulli Equation is correct.

(b) (20) Stagnation (10) pts streamline (10) pts must be drawn correctly. If vague “along unspecified streamline”, then (10/20). If word “streamline” present anywhere in solution then (5/10).

(c) (35) Result; idea (10) pts, execution (20) pts, result (5) pts.

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