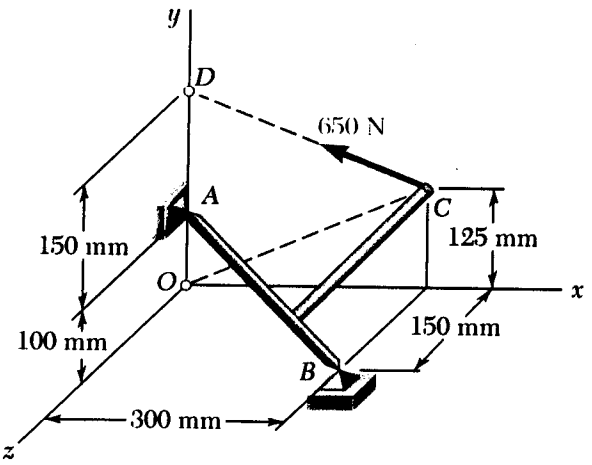
ME C85/CE C30 Mid-Term Exam #1, Fall 2012 *Name:* Shashank Nawathe

Friday, February 24, 2012, 12 –1 PM.

Please write your name at the top of each page as indicated and write all answers in the space provided. If you need additional space, write on the back sides. Do not remove or add any pages. **Assume all problems are two-dimensional unless noted otherwise. For all answers, where appropriate, provide units.** *Good luck!*

PROBLEM 1: 30 pts total

For this three-dimensional problem, two rods are welded together to form a T-shaped lever that is acted upon by a 650-N force as shown. Note that pt C is located in the x-y plane, and point A is on the y-axis



i) 10 pts Express the force as a 3D vector and determine its x, y, and z components.

$\bar{r}_{CD} = -300\,\hat{i} + 125\,\hat{j}$; $\bar{\alpha}_{CD}$ = unit vector $= \dfrac{-300\hat{i} + 125\hat{j}}{\sqrt{300^2 + 125^2}}$

$$\bar{\alpha}_{CD} = \frac{-300\,\hat{i} + 125\,\hat{j}}{325}$$

$$\underline{F} = 650\,(\bar{\alpha}_{CD}) = 650\left(\frac{-300\,\hat{i} + 125\,\hat{j}}{325}\right)$$

$$\boxed{\underline{F} = -600\,\hat{i} + 250\,\hat{j}\ \text{N}}$$

$F_x = -600$ N ; $F_y = 250$ N ; $F_z = 0$ N

ii) 10 pts Determine the magnitude of the moment of the force about pt A.

$\bar{r}_{AC} = 300\,\hat{i} + 25\,\hat{j}$

$\bar{M}_A \quad r_{AC} \times \bar{F} = (300\,\hat{i} + 25\,\hat{j}) \times (-600\,\hat{i} + 25\,\hat{j})$ N mm

$= 90{,}000\,\hat{k}$ N mm

$\bar{M}_A = 90\ \text{N m}\ \hat{k}$ $\qquad$ $\boxed{\text{Magnitude} = 90\ \text{N m}}$

iii) 5 pts What is the component of this moment along the rod AB?

$M_{AB} = \bar{M}_A \cdot \bar{\alpha}_{AB}$

$\bar{r}_{AB} = 300\,\hat{i} - 100\,\hat{j} + 150\,\hat{k}$; $\bar{\alpha}_{AB}$ = unit vector along

$$= \frac{300\,\hat{i} - 100\,\hat{j} + 150\,\hat{k}}{\sqrt{300^2 + 100^2 + 150^2}}$$

$$\bar{\alpha}_{AB} = \frac{300\,\hat{i} - 100\,\hat{j} + 150\,\hat{k}}{350}$$

$$M_{AB} = \frac{90000 \times 150}{350}\ \text{N mm} = 38.6\ \text{Nm}$$

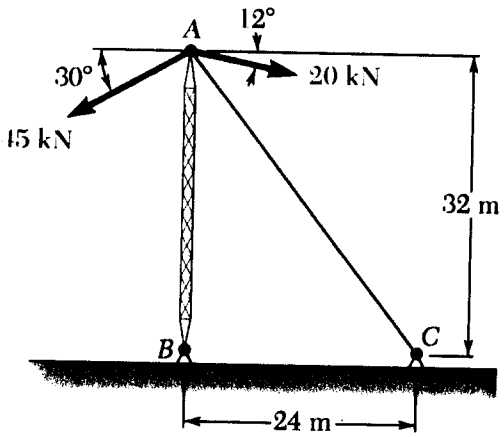
$$\boxed{M_{AB} = 38.6\ \text{Nm}}$$

iv) 5 pts Provide a physical interpretation for that component.

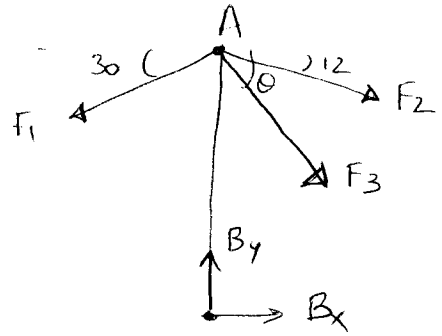
M_{AB} is that moment which is acting to twist the whole structure around the axis along rod AB

PROBLEM 2: 25 pts total

Plyon AB is attached to the ground at B via a hinge joint, and has three cables attached at top A. Two of the cables exert tensile forces as shown; a third cable AC is also attached, as shown, with unknown tension. Ignoring the mass of the pylon and cables, determine the tension force in cable AC if the two-force member AB is to remain vertically oriented as shown.



FBD :



Taking moment about A $\quad \Sigma M_A = B_x(32) = 0$

$$B_x = 0$$

$$\Sigma F_x = 0 \Rightarrow B_x + F_2 \cos 12 + F_3 \cos\theta = F_1 \cos 30$$

$$\tan\theta = 32/24 = 4/3 \Rightarrow \cos\theta = 3/5$$

$$\because B_x = 0, \quad F_2 \cos 12 + \frac{3F_3}{5} = F_1 \cos 30$$

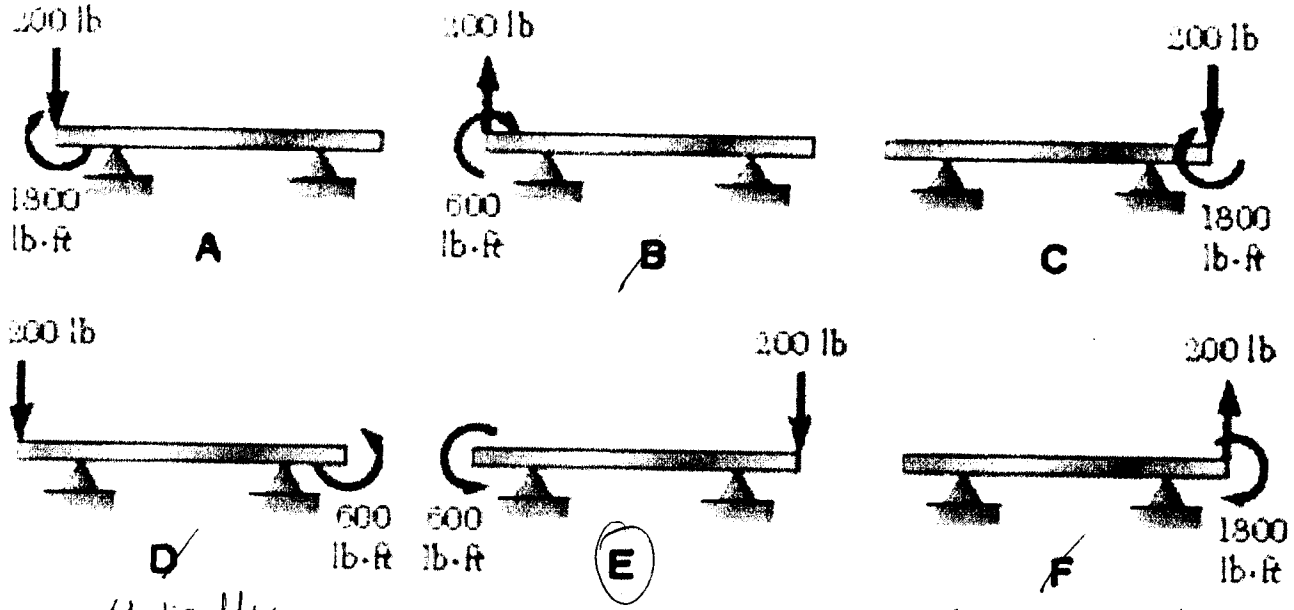
$$-5/3\,(20\cos 12 - 45\cos 30) = F_3$$

$$F_3 = 32.34 \text{ KN}$$

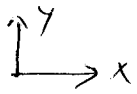
Tension in cable AC = 32.34 KN

PROBLEM 3: 20 pts total

A beam that is 12 ft long is loaded in various ways as shown. Which single loading configuration B–F is statically equivalent to the loading configuration A? *Don't just guess: for full credit you need to show your work!*



Evaluating statically equivalent Force-couple systems about the leftmost point of the beam for all the loading configurations B–F

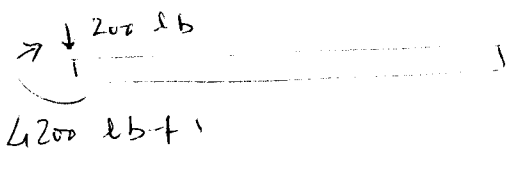


B

$F = 200$ lb

$M = 600$ lb-ft

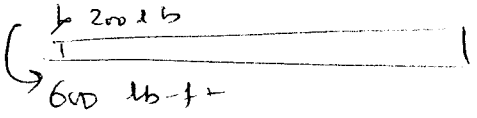
C



$F = -200$ lb

$M = 200 \times 12 + 1800 = 4200$ lb-ft

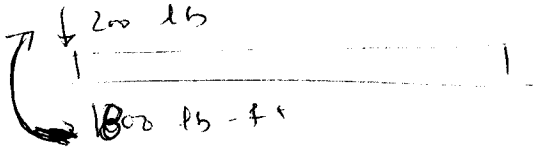
D



$F = -200$ lb

$M = -600$ lb-ft

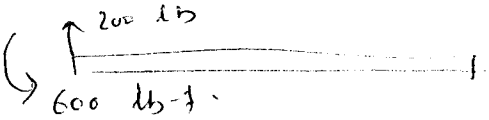
E



$F = -200$ lb

$M = 200 \times 12 - 600 = 1800$ lb-ft

F



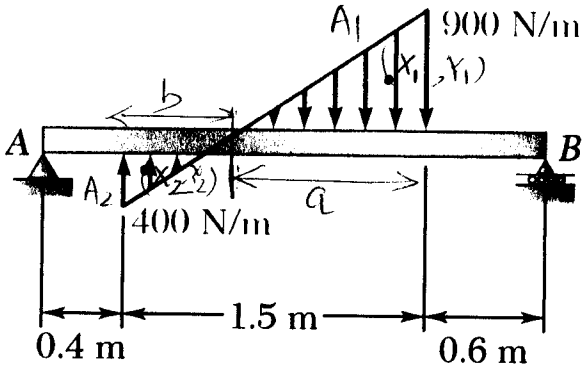
$F = +200$ lb

$M = -200 \times 12 + 1800 = -600$

E is equivalent to A

PROBLEM 4: 25 pts total

i) 15 points What is the horizontal location X* of the single resultant force of the distributed loading? Assume X=0 at the left end of the beam.



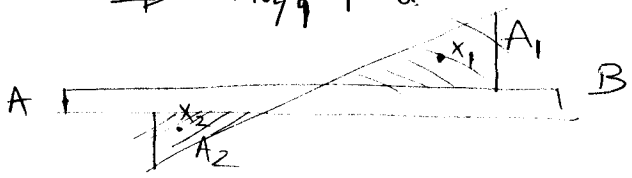
$a + b = 1.5$

$900/a = 400/b$

$b = \frac{4a}{9}$

$\Rightarrow 4a/9 + a = 1.5 \Rightarrow a = \frac{9}{13}(1.5) = 1.038$

$b = (4/13)(1.5) = 0.462$



Considering A as Origin.

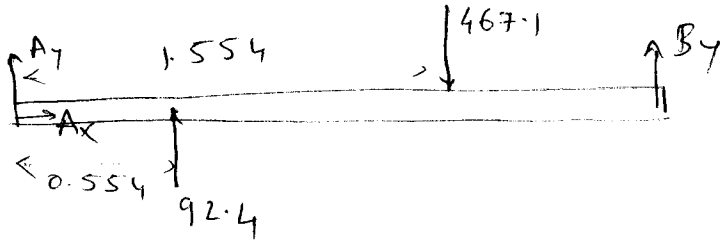
$X_1 = 1.9 - a/3 = 1.9 - \frac{1.038}{3} = 1.554$

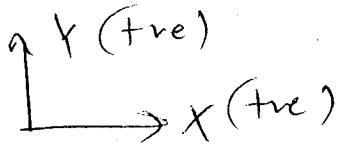
$X_2 = 0.4 + b/3 = 0.4 + \frac{0.462}{3} = 0.554$

$A_1 = \frac{1}{2}(900)(a) = 467.1$; $A_2 = \frac{1}{2}(400)(b) = 92.4$; $x^* = \frac{A_1 X_1 + A_2 X_2}{A_1 + A_2} = 1.8$

$\boxed{x^* = 1.5}$

ii) 10 points Ignoring the mass of the beam, what are the horizontal (X, positive to the right) and vertical (Y, positive upwards) components of the reaction forces at A and B?





$\Sigma M_A = 0 \Rightarrow B_y(2.5) = (467.1)(1.554) - (92.4)(0.554)$

$\boxed{B_y = 269.88\ N}$

$\Sigma F_x = 0 \Rightarrow \boxed{A_x = 0}$

$\Sigma F_y = 0 \Rightarrow A_y = -467.1 + 92.4 + 269.88$

$\Rightarrow \boxed{A_y = -105.18\ N}$

$\underline{A} = -105.18\ \hat{j}\ N \qquad \underline{B} = 269.88\ \hat{j}\ N$