<u>Reminder</u>

$$\tau = rF\sin\theta \; ; \; L = rp\sin\theta \; ; \; I_{rod/center} = \frac{1}{12}ML^2 \; ; \; I_{solid\,sphere/diameter} = \frac{2}{5}MR^2 \; ;$$

$$I_{solid\,cylinder/axis} = \frac{1}{2}MR^2 \; ; \; I_{hollow\,sphere/diameter} = \frac{2}{3}MR^2 \; ; \; I_{hollow\,cylinder/axis} = MR^2$$

Problem 1 – Spacecraft around Earth (25 pts)

A spacecraft of mass m is in circular orbit above the equator at distance d above Earth's surface. Earth has mass M and radius R, and air resistance neglected.

a- Determine the moment of inertia of the spacecraft about the Earth's South-North axis while it is in orbit if the spacecraft can be approximated as a solid sphere with a radius r_{sc}. Show that the spacecraft can actually be considered as a point particle.

b- How much work needs to be done against the gravitational force to bring the spacecraft from its original distance to infinity?

c- What is the minimum speed v_e that the spacecraft needs to reach from its original altitude in order to escape the gravitational field of the Earth?

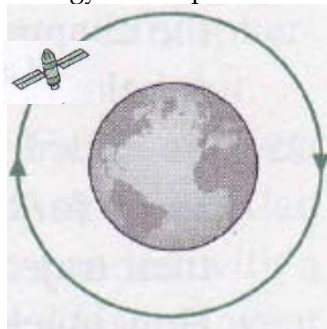d- What is the sign of the mechanical energy of the spacecraft when $v > v_e$? Explain why.

Figure 1

Problem 2 – Ballistic pendulum (25 pts)

An apple of mass M suspended from a massless string is used as a ballistic pendulum to determine the speed v_i of an arrow of mass m that hits it, as shown in Figure 2. Air resistance is neglected, and we'll assume that the arrow hits the apple horizontally, the apple being initially at rest.

a- From the height h of the swing, determine the speed v_f of the combined system right after the collision. What is the direction of $\vec{v}_f$?

b- Calculate the speed v_i of the arrow immediately before it strikes the apple.

c- Name what type of collision this is; you will need to show a proof to justify your claim.

d- Afterwards, an experiment is performed where air resistance reduces the maximum height reached by the apple and arrow to H. Calculate the work done by the drag force on the ballistic pendulum from the collision until the arrow and apple reaches this new maximum height H.
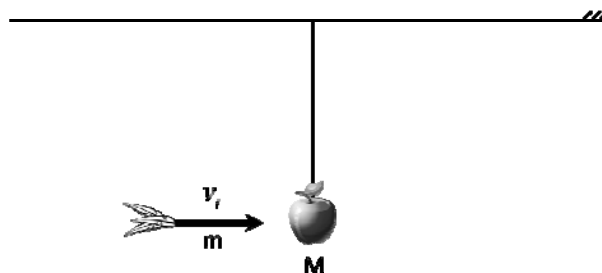

Figure 2

Problem 3 – Ball race (25 pts)

A hollow ball and a solid ball of same radius r and same mass M are simultaneously released from the top (height h) of a double-track loop-the-loop (two different tracks that are right next to each other). They are rolling without slipping due to friction.

a- Explain why mechanical energy conservation can still be applied, in spite of the friction force experienced by the balls.
b- What is the change in kinetic energy of the balls from the beginning to the end of the track?
c- Assuming that they both make it through the loop without falling off the track, which one of the two balls wins the race?
d- What is the minimum initial height h necessary to enable the slowest ball to make it through the loop?
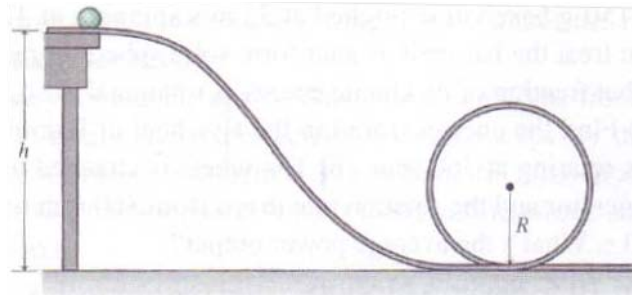

Figure 3

Problem 4 – Rope course (25 pts)

One of the aerial bridges of a rope course is made of two massless ropes, each of length $L/2$, attached to a uniform board of mass M and length L. The left and right ropes make an angle α and β respectively with the vertical trees they are attached to, as shown on Figure 4 (note $\alpha \neq \beta$).

a- A person of mass m steps on the left end of the board and then stops. What are the forces experienced by the board? Draw a freebody diagram with the forces in the appropriate locations where they're acting.
b- For the board to be in static equilibrium, what are the necessary conditions that need to be satisfied?
c- Calculate the tension forces exerted by the ropes on the board in terms of the given variables.
d- Describe qualitatively what happens to the tension force in the two ropes (i.e. gets bigger, smaller, stays the same) as the person walks toward the right end of the board. Explain your reasoning.
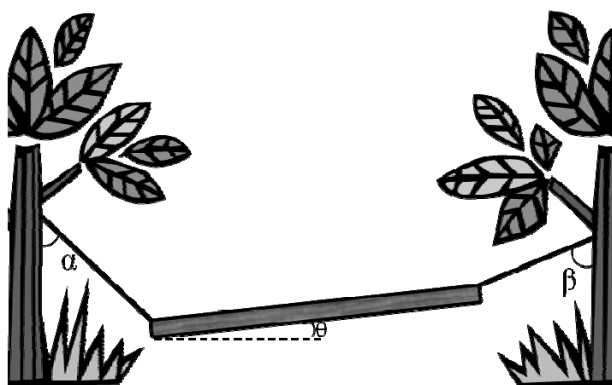

Figure 4