Angela Wu
SID: 15947894
(Monday Section)

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Stat 25 Fall 2003
Midterm 1
Open book, 50 minutes

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Note: *Keep 3 decimal places in calculations and in answers. Frame your answers. Show all work for full credit.*

Problem 1. *(40 minutes, 40 points)*
A wholesaler sells blank recordable compact discs (CD-R) to a retailer by boxes of 20 discs for $5 a box. If a box has at least one defective disc, the retailer can return the whole box to the wholesaler and get a $8 refund. Chance of a disc to be defective is 1%. The retailer just takes delivery of 15 boxes.
A. What is the chance that a given box will have exactly 2 defective discs?
B. What is the chance that exactly 3 boxes will have to be returned?
C. Suppose the retailer is inspecting a box that has 3 defective discs. The retailer has time to check only 5 discs. If the 5 discs are chosen at random without replacement from the box, what is the chance that the retailer will find at least one defective disc?
D. What is the expected revenue of the wholesaler for the sale of the 15 boxes? (revenue is total sale amount minus the eventual refunds)

A) X = # of defective discs. $X \sim \text{bin}(20, 0.01)$

$P(X=2) = \binom{20}{2}(0.01)^2(0.99)^{20-2}$ $\left(= \binom{n}{x} p^x (1-p)^{n-x}\right)$

$= 190 \times 0.0001 \times 0.835$

$= 0.016$ ANS (to 3dp)

B) X = # defective discs. $X \sim \text{bin}(20, 0.01)$

$P(X \geq 1) = 1 - P(X=0) = 1 - \binom{20}{0}(0.01)^0(0.99)^{20} = 1 - (1 \times 1 \times 0.818)$

$= 1 - 0.818$

$= 0.182$ (to 3dp) ← this is the prob. that any given box will be returned.

Y = # returned. $Y \sim \text{bin}(15, 0.182)$

$P(Y=3) = \binom{15}{3}(0.182)^3(1-0.182)^{15-3}$

$= 455 \times 0.006 \times 0.089$

$= 0.246$ ANS (to 3dp)

C) A = # defective discs picked out of a box with 3 defective discs $\Rightarrow A \sim H(5, 20, 3)$

$P(A \geq 1) = 1 - P(A=0) = 1 - \dfrac{\binom{3}{0}\binom{20-3}{5-0}}{\binom{20}{5}} = 1 - \dfrac{\binom{3}{0}\binom{17}{5}}{\binom{20}{5}} = 1 - \dfrac{1 \times 6188}{15504}$

$= 1 - 0.399 = 0.601$ ANS. (to 3dp)

-0

PTO

D) R = total revenue B = boxes sold Y = boxes returned.

$R = 5B - 8Y$

In this case, $B = 15$

$R = 75 - 8Y$

We know $Y \sim bin(15, 0.182)$

$E(Y) = np = 15 \times 0.182$

$= 2.73$

$E(R) = E(75 - 8Y)$

$E(aX+b) = aE(X)+b$

So, $E(R) = -8E(Y) + 75$

$= -8 \times 2.73 + 75$

$= \$53.16$ ANS

Problem 2. *(10 minutes, 10 points)*
Assembly of an optical data storage product requires optical alignment of the parts. Often several trials are required before a good alignment is obtained. Chance of a successful alignment is 0.9. Assume trials are independent.

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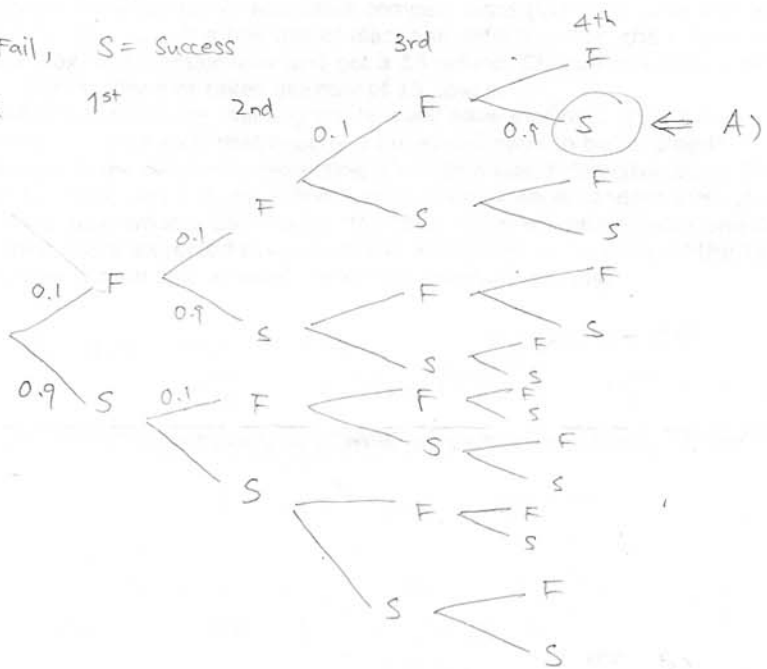
A. What is the probability that the first successful alignment requires exactly 4 trials?
B. What is the probability that the first successful alignment requires at least 4 trials?

$P(\text{Success}) = 0.9$ $\quad P(A|B) = P(A)$

F = Fail, S = Success



A) $P(F,F,F,S) = 0.1 \times 0.1 \times 0.1 \times 0.9 = 0.0009$ good

(∵ since events independent, $P(A \cap B) = P(A) \times P(B)$)

$= \boxed{0.001}$ ANS (to 3dp.)

B) $P(\#F \geq 3) = 1 - P(F \leq 2) = 1 - [P(S) + P(F,S) + P(F,F,S)]$

$= 1 - (0.9 + 0.1 \times 0.9 + 0.1^2 \times 0.9)$

$= 1 - 0.999$

$= \boxed{0.001}$ ANS