Fall 2009

# Math 53 Final Exam

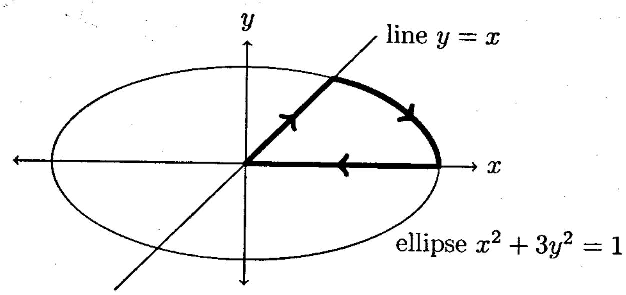
Prof [signature]

1. (14 points) Compute the line integral $\int_C y\,dx$ for these curves $C$:

   (a) $C$: $x(t) = \cos t$, $y(t) = \frac{1}{\sqrt{3}} \sin t$, $0 \le t \le 2\pi$.

   (b) $C$ is the ellipse $x^2 + 3y^2 = 1$ with clockwise orientation.

   (c) $C$ is the bolded curve drawn below:




2. (12 points) Let $C$ be a curve in the plane that goes from $(0,0)$ to $(\frac{\pi}{2}, \pi)$. Is the line integral

$$\int_C \cos x \cos y\,dx + \sin x \sin y\,dy$$

path independent? If it is, compute it. Now answer the same question for

$$\int_C \cos x \cos y\,dx - \sin x \sin y\,dy$$

3. (12 points) $C$ is square with vertices $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$ with counterclockwise orientation. For differentiable $P(x,y)$, show that

$$\int_C P\,dx = -\int_0^1 \int_0^1 P_y(x,y)\,dx\,dy$$

4. (12 points) For the same $C$ as in problem 3., evaluate the line integral

$$\int_C (5 - xy - y^2)\,dx + (x^2 - 2xy)\,dy.$$

5. (12 points)

   (a) Compute the upward unit normal of the surface $z = xy$.

   (b) Compute the area $A$ of this surface inside the cylinder $x^2 + y^2 = R^2$, where $R$ is a positive constant. What is $\lim_{R \to 0} \frac{A}{\pi R^2}$?

6. (12 points) $S$ is the portion of $z = xy$ inside the cylinder $x^2 + y^2 = 1$. Compute

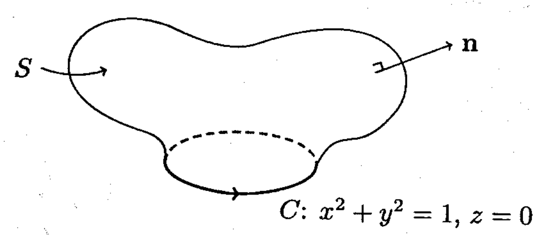
$$\iint_S (y\mathbf{i} + x\mathbf{j} + \mathbf{k}) \cdot \mathbf{n}\,dS$$

7. (12 points) Compute the surface integral of the vector field $\mathbf{F} = x\mathbf{i} + y\mathbf{j}$ over the hemisphere $x^2 + y^2 + z^2 = 1$, $z > 0$. (Hint: First, consider the *closed* surface made of the hemisphere and the disk $x^2 + y^2 \leq 1$ in the $xy$-plane).

8. (14 points) The vector field $\mathbf{F}$ is the curl of another vector field $\mathbf{B}$, $\nabla \times \mathbf{B} = \mathbf{F}$.

   (a) Compute $\nabla \cdot \mathbf{F}$.

   (b) Let $S$ be a surface with its perimeter $C$ the unit circle in the $xy$-plane, like this:




Take $\mathbf{B} = -y\mathbf{i} + x\mathbf{j}$. Compute $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$, with $\mathbf{n}$ oriented as shown.