MATH 54, 2nd midterm test.

All the necessary work to justify an answer and all the necessary steps of a proof must be shown clearly to obtain full credit. Partial credit **may** be given but only for significant progress towards a solution. Show all relevant work in logical sequence and indicate all answers clearly. Cross out all work you do not wish considered. 1 page of notes is allowed. Books and electronic devices are not allowed during the test.

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1. (8 pp.) Let S be the set of all polynomials f in $\mathbb{P}_3$ such that $f(0) = f(2)$. Show that S is a subspace of $\mathbb{P}_3$ and find a basis of S.

2. (8 pp.) Is the linear system

$$\begin{aligned} x_1 + x_2 &= 1 \\ x_1 + x_2 &= 3 \\ x_1 + x_3 &= 8 \\ x_1 + x_3 &= 2 \end{aligned}$$

solvable? Describe all its least-squares solutions.

3. (8 pp.) Show that the formula

$$\langle f, g \rangle := \int_0^1 f(t)g(t)dt$$

defines an inner product on $C[0,1]$, the vector space of all continuous function on the interval $[0,1]$. Using Gram-Schmidt orthogonalization, find an orthonormal basis for the subspace spanned by 1, $t-1$ and t^2+t.

4. (8 pp.) (a) Given an $n \times n$ orthogonal matrix U, show that

$$\|U\mathbf{x}\| = \|\mathbf{x}\| \quad \text{for all} \quad \mathbf{x} \in \mathbb{R}^n.$$

(b) Let A be a symmetric $n \times n$ matrix whose largest eigenvalue is λ_{max}. Using the result of part (a), show that

$$\mathbf{x}^T A\mathbf{x} \leq \lambda_{max} \qquad \text{for all} \quad \mathbf{x} \in \mathbb{R}^n \quad \text{such that} \quad \|\mathbf{x}\| = 1.$$