University of California, Berkeley
Fall Semester 2011

Department of Mechanical Engineering
Instructors: A.K. Packard, P. Papadopoulos

Midterm Examination 2

November 4, 2011

NAME : _____

SID : _____

SECTION : **1** or **2** (please circle your lecture section)

LAB :

#11: TuTh 8-10	#12: TuTh 10-12	#13: TuTh 12-2	#14: TuTh 2-4
#15: TuTh 4-6	#16: MW 8-10	#17: MW 10-12	#18: MW 2-4
#19: MW 4-6	#20: TuTh 10-12	#21: MW 3-5	#22: TuTh 4-6

(please circle your lab section)

Part	Points	Grade
A	4	
B	8	
C	3	
D	8	
E	9	
F	8	
G	5	
TOTAL	**45**	

Notes:
1. Write your name on the top right corner of each page.
2. Record your answers only in the spaces provided.
3. You may not ask questions during the exam.
4. You may not leave the exam room before the exam ends.

 Name: _____

Part A (4 points)

A.1 (2 points)

The following code uses the array M defined as:

$$M = \begin{bmatrix} 1 & 3 & -2 \\ 7 & -5 & 1 \end{bmatrix}$$

```
M = [1 3 -2;7 -5 1];
temp = 0;
for k=M
   temp = temp + k(2);
end
temp
```

Record the output of the MATLAB code given above.

```
temp = 3
```

A.2 (2 points)

The following code uses the array B defined as:

$$B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

```
B = [1 2;3 4];
temp = 0;
for k=B
   for j=B
      temp = temp + k'*j;
   end
end
temp
```

Record the output of the MATLAB code given above.

```
temp = 58
```

 Name: _____

Part B (8 points)

Consider a sequence of positive integer numbers defined by $a(1) = 1, a(2) = 1$, and $a(3) = 2$, and the recursive formula

$$a(n+3) \;=\; a(n+2) + a(n+1) + a(n) \quad , \quad n = 1, 2, \ldots$$

B.1 (4 points)

Using recursion, complete the function `card` that computes the value of the k-th term in this sequence.

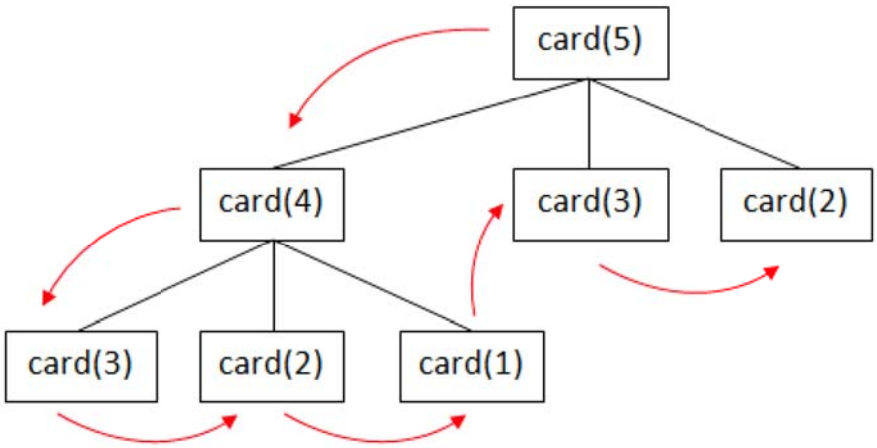
```
function a = card(k)
% Computes the value of the k-th term
% of the sequence of integers defined above
% where k is a positive integer

disp(['card, k = ' int2str(k)]);

if k==3

  a = 2;

elseif k == 1 || k == 2

  a = 1;

else

  a = card(k-3) + card(k-2) + card(k-1);

end
```

B.2 (2 points)

With the function `card` properly implemented, the following code is executed.

```
>> c5 = card(5);
```

Draw the tree depicting all the recursive calls made by the `card` function. Also, include the arrows showing the pre-order traversal of the tree.



B.3 (2 points)

With the function `card` properly implemented, the following code is executed.

```
>> c5 = card(5);
```

Carefully write what is displayed on the screen.

card, k = 5
card, k = 4
card, k = 3
card, k = 2
card, k = 1
card, k = 3
card, k = 2

 Name: _____

Part C (3 points)

The following lines of code are executed in the sequence listed:

```
>> h = plot(1:10,cos(1:10));

>> set(h,'LineWidth',2.7);

>> h2 = h;

>> clear h;

>> get(h2,'LineWidth');

>> get(h, 'LineWidth'); This line will produce error 4.

>> h3 = h2;

>> delete(h2);

>> get(h3,'LineWidth'); This line will produce error 2.

>> get(h2,'LineWidth'); This line will produce error 2.
```

At least one of these lines of code WILL produce an error. **Circle the error-producing line(s) of code.** Listed below are several approximate error messages, with the variable or function names replaced by `[name]`. Next to each of the error-producing line(s) above, write the number corresponding to the error it will produce. **The circle(s) and error number(s) you write down must be legible**.

1. `The expression to the left of the equals sign is not a valid target for an assignment.`
2. `Invalid handle object.`
3. `Undefined function or method [name] for input arguments of type double`
4. `Undefined function or variable [name]`
5. `Subscript indices must either be real positive integers or logicals.`

Part D (8 points)

D.1 (4 points)

The number $(0.101)_2$ employs base2 notation, and is equal to $1 \cdot \frac{1}{2} + 0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{8} = 0.625$.

Suppose the number R satisfies $0 \leq R < 1$ and is an integer multiple of $\frac{1}{32}$. Let $(0.B)_2$ be the base2 representation of R. For example the number $(0.10100)_2$ employs base2 notation, and is equal to $1 \cdot \frac{1}{2} + 0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{8} + 0 \cdot \frac{1}{16} + 0 \cdot \frac{1}{32} = 0.625$. The fraction B is written as a 1-by-5 array, `[1 0 1 0 0]`.

Complete the code below to compute the fraction B. Use the variable `R` to represent a given number for which the conversion is taking place.

```
B = zeros(1,5);

for k = 1:5

  if R >= (1/2)^k

    B(k) = 1;

    R = R - (1/2)^k;

  end

end
```

What is the value of `B` after execution of this code if the initial value of `R` is equal to 0.8125?

`B = 1 1 0 1 0`

D.2 (1+1 points)

Record the output of each of the following MATLAB commands:

```
>> r = 1000*rand;
>> r + eps(r) == r

ans = 0


>> 10 + eps(1) == 10

ans = 1
```

D.3 (1+1 points)

The IEEE `double` representation for $x = 17.125$ is

$$\mathtt{S} = 1,\ \mathtt{E} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & \underbrace{\cdots}_{0_{1\times 42}} & 0 \end{bmatrix}$$

(note that the $\cdots$ represent 42 bits of value 0, so that F in total is 52 bits).

What is the IEEE `double` representation for $y = 34.25$?

Answer: `S` and `F` unchanged

$$\mathtt{S} = 1,\ \mathtt{E} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & \underbrace{\cdots}_{0_{1\times 42}} & 0 \end{bmatrix}$$

What is the IEEE `double` representation for $z = 25.125$?

Answer: `S` and `E` unchanged

$$\mathtt{S} = 1,\ \mathtt{E} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & \underbrace{\cdots}_{0_{1\times 42}} & 0 \end{bmatrix}$$

Part E (9 points) The polynomial of degree $n-1$ passing through points (x_1, y_1), (x_2, y_2), $\ldots$, (x_n, y_n) is given by the equation

$$P(x) = \sum_{i=1}^{n} L_i y_i \,,$$

where

$$L_i = \frac{(x - x_1)\ldots(x - x_{i-1})(x - x_{i+1})\ldots(x - x_n)}{(x_i - x_1)\ldots(x_i - x_{i-1})(x_i - x_{i+1})\ldots(x_i - x_n)} \,.$$

Complete the following MATLAB code that determines the value of the polynomial $P(x)$ evaluated at a given set of values.

```
function yValues = myLagrangeInt(Xd, Yd, xValues)
% Determines the polynomial function that passes through
% a given set of points in the (x,y) plane
% Input variables:
%   Xd: 1-by-n, x-coordinates of the interpolated points
%   Yd: 1-by-n, y-coordinates of the interpolated points
%   xValues: vector of input values
% Output variable:
%   yValues: vector of values of P for the corresponding xValues


nD = length(Xd);

nValues = length(xValues);
yValues = zeros(nValues, 1)

for i=1:nD

   L = ones(size(yValues));

   for j=1:nD

      if  i ~= j

         L = L.*(( xValues - Xd(j) ) / ( Xd(i) - Xd(j) ));

      end

   end

   yValues = yValues + Yd(i) * L ;

end
```

Part F (8 points)

A simple variant of Newton's method detects that the sequence of approximate solutions $x^{(1)}, x^{(2)}, \ldots$ is diverging, and restarts the iteration with an alternate value. In this problem, diverging is defined as meaning that the current iterate, $x^{(k)}$, satisfies the condition $|f(x^{(k)})| > divtol$, where $divtol$ is a given positive scalar. If divergence is detected, then the method restarts the iteration from a new initial guess $(1 + \text{rand}) * x^{(0)}$, where $x^{(0)}$ was the original initial guess.

Complete the following non-recursive function, which implements this variant of Newton's method:

```
function x = newton(fhan,fdhan,x0,tol,divtol)
% Newton's method to find roots of a function f
% Input Arguments
%  fhan: function handle to f
%  fdhan: function handle to the derivative of f
%  x0: initial guess of root
%  tol:  tolerance for iteration converge (in x)
%  divtol: tolerance for divergence check
% Output Argument
%  x: final estimate of root

abs_error = 2*tol;
x = x0;

while (abs_error>tol)

   % Calculate next iterate of Newton's method

   x_next =   x - fhan(x) / fdhan(x);

   abs_error = abs(x_next-x);

   x = x_next
   % Check for divergence

   if  abs(fhan(x)) > divtol
      % Restart with new initial guess

      x0 =  (1 + rand)*x0;

      abs_error = 2 * tol;

   end
end
```

Part G (5 points)

In this problem we determine the coefficients of a quadratic function of the form:

$$p(x) = c_1x^2 + c_2x + c_3$$

The polynomial $p(x)$ **must** satisfy the constraint $p(1) = 0$. We would also like $p(x)$ to satisfy the 4 constraints below.

$$p(-1) = 5, \quad p(0) = 10, \quad p(2) = 1, \quad p(3) = 12.$$

However, this is not possible, since the system of equations is over-determined. Instead, we wish to minimize the error,

$$E = (p(-1) - 5)^2 + (p(0) - 10)^2 + (p(2) - 1)^2 + (p(3) - 12)^2$$

using least squares to solve this minimization.

G.1 (1 point)

By hand, solve for c_3 in terms of c_1 and c_2 such that the constraint, $p(1) = 0$, holds.

$$p(1) = c_1 + c_2 + c_3 = 0$$
$$c_3 = -c_1 - c_2$$

G.3 (1 point)

Rewrite the defining equation for p by substituting in the value for c_3 obtained in Part G.1. The rewritten $p(x)$ should now only contain the c_1 and c_2 coefficients.

$$p(x) = c_1(x^2 - 1) + c_2(x - 1)$$

G.3 (3 points)

Set up the Least Squares problem which minimizes the error E

$$\min_c \|Ac - b\|$$

by correctly defining the matrix A and vector b, where $c = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$.

$$A = \begin{bmatrix} 0 & -2 \\ -1 & -1 \\ 3 & 1 \\ 8 & 2 \end{bmatrix} \qquad b = \begin{bmatrix} 5 \\ 10 \\ 1 \\ 12 \end{bmatrix}$$