(1) (40pts)

Given the nonlinear system

$$\frac{dx}{dt} = 1 - xy \qquad\qquad \frac{dy}{dt} = x - y^3$$

(a) Find all the critical points of the system

(b) Chose any one of the critical points in part (a) and find the corresponding linear system near that critical point

(c) Find the eigenvalues of the corresponding linear system in part (b)

(d) Based on your result from part (c), discuss the nature and stability of the nonlinear system

Hint: Translate axes by the substitutions $X = x - h$, $Y = y - k$, where h and k are chosen so that the critical point becomes $(0,0)$

(2) (40pts)

(a) Derive the expression for the Laplace transform of e^{at}

(b) Use the Laplace transform to solve the initial value problem:

$$y''(t) + y(t) = \sin 2t \; ; \qquad y(0) = 2, \quad y'(0) = 1$$

(3)   (20pts)

Determine the Fourier series of the periodic square-wave function

$$f(x) = \begin{cases} -K & -L \le x < 0 \\ K & 0 \le x < L \end{cases}$$