EECS 20N: Structure and Interpretation of Signals and Systems MIDTERM 1
Department of Electrical Engineering and Computer Sciences 20 September 2011
University of California Berkeley

LAST Name ____________________ FIRST Name ____________________

Lab Time ____________________

- **(10 Points)** Print your name and lab time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 70 minutes to complete. You will be given at least 70 minutes, up to a maximum of 80 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except one double-sided 8.5" $\times$ 11" sheet of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 6.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the six numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

MT1.1 (20 Points) Consider a continuous-time signal x described by $x(t) = e^{i2\pi f_0 t}$, for all t, where f_0 is in cycles per second (Hertz). The signal x is called a *pure tone*, because f_0 is the only frequency present in it. We represent the frequency content of x (i.e., its "spectrum") by $\delta(f - f_0)$, where the impulse is a Dirac delta.

(a) (10 Points) You know that for the continuous variable f, and a non-zero constant α, the following relation holds:

$$\delta(\alpha f) = \frac{1}{|\alpha|}\,\delta(f).$$

Show that

$$\delta(\alpha(f - f_0)) = \frac{1}{|\alpha|}\,\delta(f - f_0).$$

(b) (10 Points) Use the result of part (a) to provide a well-labeled plot of the spectrum of x along the radial-frequency axis (i.e., the ω-axis, where the frequency variable ω is in radians per second).

You must first express $\delta(f - f_0)$ in terms of an appropriate Dirac delta written in terms of the frequency variable ω. Recall that $\omega = 2\,\pi\,f$, and define $\omega_0 = 2\,\pi\,f_0$.

MT1.2 (45 Points) In this problem, we look at the roots of the quadratic polynomial

$$Q(z) = a\,z^2 + b\,z + c,$$

where the coefficients satisfy $a > 0$, $b \geq 0$, and $c \geq 0$.

The two roots are given by

$$z_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad z_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

Note that we may rewrite the polynomial as

$$\begin{aligned} Q(z) &= a\,(z - z_1)(z - z_2) \\ &= a\left[z^2 - (z_1 + z_2)\,z + z_1\,z_2\right], \end{aligned}$$

which implies that the sum of the roots is $z_1 + z_2 = -\dfrac{b}{a}$ and their product is $z_1\,z_2 = \dfrac{c}{a}$.

(a) (10 Points) Show that, depending on the coefficient values, (I) each of the two roots z_1 and z_2 is real (i.e., $z_1, z_2 \in \mathbb{R}$), OR (II) the two roots form a complex-conjugate pair (i.e., $z_2 = z_1^* \notin \mathbb{R}$).

If the coefficients a, b, and c are such that the two roots z_1 and z_2 are complex-valued, express each root in Cartesian (rectangular) form (in terms of the coefficients).

(b) (35 Points) Keep the coefficients b and c fixed (and positive), and let the coefficient a vary from 0^+ (i.e., infinitesimally small positive value) toward $+\infty$.

(i) (20 Points) Prove that corresponding to the range of values of the coefficient a for which the two roots are complex-valued, each of them satisfies the constraint

$$\left| z_\ell + \frac{c}{b} \right| = \frac{c}{b}, \quad \ell = 1, 2.$$

Hint: Eliminate the coefficient a by combining the expressions for $z_1 + z_2$ and $z_1 \, z_2$ in just right way. Then, exploit the fact that $z_2 = z_1^*$.

(ii) (15 Points) Provide, *on the complex plane*, a well-labeled diagram of the path(s) along which the two roots z_1 and z_2 move as the coefficient a varies in the way described. Indicate the direction of the movement of each of the two roots as the coefficient a increases. Also, specify the constraints on the coefficients a, b, and c at any point where the roots transition into, or out of, the real axis, as well as where the roots tend to as $a \to +\infty$. Use the result of part (i), even if you could not prove it. *A diagram without a valid explanation will receive no credit.*

MT1.3 (40 Points) Parts (a) and (b) of this problem are independent, so you may approach them in either order.

(a) (25 Points) Consider the continuous-time signals x_1 and x_2 described by $x_1(t) = \cos(2\pi t)\, u(t)$ and $x_2(t) = \sin(2\pi t)\, u(t)$, for all t. The function u is the continuous-time unit step. These signals are called a *tone bursts*, in part because they represent suddenly-applied sinusoids.

(i) (10 Points) Provide well-labeled plots of the signals x_1 and x_2. Explain how you label the important features of your plots.

(ii) (15 Points) Determine, and provide well-labeled plots of, $\dot{x}_1(t)$ and $\dot{x}_2(t)$, the first-order time derivatives of the signals x_1 and x_2.

(b) (15 Points) Consider the continuous-time signal r described by $r(t) = \cos(\theta(t))$, where $\theta(t) = 2\pi t^2$, for all t. Provide a well-labeled plot of $r(t)$, for all t. Recall that the instantaneous frequency ω is given by $\omega(t) = \dot{\theta}(t) \stackrel{\triangle}{=} \dfrac{d\theta(t)}{dt}$, for all t, so be sure to explain how the instantaneous frequency influences your plot.

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Problem	Points	Your Score
Name	10	
1	20	
2	45	
3	40	
Total	**115**	